Chapter 5

PROBABILITY DENSITIES

5.1

$$f(x) = \begin{cases} 2e^{-2x} & \text{for } x > 0 \\ 0 & \text{elsewhere} \end{cases}$$

Since e^{-2x} is always positive, $f(x)$ is always ≥ 0.

$$\int_{-\infty}^{\infty} f(x)dx = -e^{-2x}\Big|_0^{\infty} = 1$$

Thus, $f(x)$ is a density.

5.2 To find k, we must integrate $f(x)$ from $x = 0$ to $x = 1$ and set it equal to 1. Thus,

$$\int_0^1 kx^3 dx = 1 \quad \text{implies} \quad kx^4/4 \,|_0^1 = 1$$

which implies $k/4 = 1$. Thus, $k = 4$.

(a) $P(.25 \leq X \leq .75) = \int_{.25}^{.75} 4x^3 dx = x^4|_{.25}^{.75} = 80/256$

(b) $P(X > 2/3) = \int_{2/3}^{1} 4x^3 = x^4|_{2/3}^{1} = 65/81$

5.3 The distribution function is given by

$$F(x) = \int_{-\infty}^{x} f(s)ds = x^4$$

(a) $P(X > .8) = 1 - F(.8) = .5904$

(b) $P(.2 < X < .4) = F(.4) - F(.2) = .024$

5.4 (a) Let X be a random variable with density $f(x)$. Then,

$$P(.2 < X < .8) = \int_{.2}^{.8} f(x)dx = \int_{.2}^{.8} xdx = x^2/2\Big|_{.2}^{.8} = (.64 - .04)/2 = .30$$

(b)

$$\begin{aligned} P(.6 < X < 1.2) &= \int_{.6}^{1.2} f(x)dx = \int_{.6}^{1} xdx + \int_{1}^{1.2} (2-x)dx \\ &= x^2/2\Big|_{.6}^{1} + (2x - x^2/2)\Big|_{1}^{1.2} = .32 + .18 = .50 \end{aligned}$$

5.5

$$\begin{aligned} F(x) = \int_{-\infty}^{x} f(s)ds &= \begin{cases} 0 & x < 0 \\ x^2/2 & 0 \le x \le 1 \\ 1/2 + [2s - s^2/2]|_1^x & 1 < x \le 2 \\ 1 & x > 2 \end{cases} \\ &= \begin{cases} 0 & x < 0 \\ x^2/2 & 0 \le x \le 1 \\ 2x - x^2/2 - 1 & 1 < x \le 2 \\ 1 & x > 2 \end{cases} \end{aligned}$$

(a) $P(X > 1.8) = 1 - F(1.8) = 1 - [2(1.8) - (1.8)^2/2 - 1] = 1 - .98 = .02$

(b) $P(.4 < X < 1.6) = F(1.6) - F(.4) = 2(1.6) - (1.6)^2/2 - 1 - (.4)^2/2 = .84$

5.6 We need to integrate $f(x)$ from $x = -\infty$ to $x = \infty$ and set it equal to 1.

$$\begin{aligned} \int_{-\infty}^{\infty} k/(1+x^2)dx &= k\int_{-\infty}^{\infty} 1/(1+x^2)dx = k \cdot \arctan x|_{-\infty}^{+\infty} \\ &= k(\pi/2 + \pi/2) = k\pi = 1 \end{aligned}$$

Thus, $k = 1/\pi$.

5.7 Let X have distribution $F(x)$. Then,

(a) $P(X < 3) = F(3) = 1 - 4/9 = 5/9 = .556$

(b) $P(4 \leq X \leq 5) = F(5) - F(4) = 4/16 - 4/25 = .09$

5.8 The density $f(x)$ is given by $f(x) = \frac{d}{dx}F(x)$. Thus, the density is

$$f(x) = \begin{cases} 8/x^3 & x > 2 \\ 0 & x \leq 2 \end{cases}$$

5.9 (a) $P(0 \leq \text{error} \leq \pi/4) = \int_0^{\pi/4} \cos x dx = \sin x|_0^{\pi/4} = \sin(\pi/4) = \sqrt{2}/2$

(b) $P(\text{phase error} > \pi/3) = \int_{\pi/3}^{\pi/2} \cos x dx = \sin(\pi/2) - \sin(\pi/3) = 1 - \sqrt{3}/2$
$= .1339$

5.10 (a) $P(\text{no. of miles tires last} \leq 10,000 \text{ miles})$

$$= \frac{1}{20}\int_0^{10} e^{-x/20} dx = -e^{-x/20}\Big|_0^{10} = 1 - e^{-1/2} = .3935$$

(b) $P(16,000 \leq \text{no. of miles tires last} \leq 24,000)$

$$= \frac{1}{20}\int_{16}^{24} e^{-x/20} dx = e^{-16/20} - e^{-24/20} = .1481$$

(c) $P(\text{no. of miles tires last} \geq 30,000)$

$$= \frac{1}{20}\int_{30}^{\infty} e^{-x/20} dx = e^{-30/20} = .2231$$

5.11 Integrating the density function by parts shows that the distribution function is given by

$$F(x) = 1 - \frac{1}{3}xe^{-x/3} - e^{-x/3}$$

Thus,

$$\begin{aligned}
&P(\text{power supply will be inadequate on any given day}) \\
&= P(\text{consumption} \geq 12 \text{ million kwh's}) \\
&= 1 - F(12) = 4e^{-4} + e^{-4} = 5e^{-4} = .0916
\end{aligned}$$

5.12 By the definition of variance

$$\begin{aligned}
\sigma^2 &= \int_{-\infty}^{\infty} (x-\mu)^2 f(x)dx \\
&= \int_{-\infty}^{\infty} x^2 f(x)dx - 2\mu \int_{-\infty}^{\infty} x f(x)dx + \mu^2 \int_{-\infty}^{\infty} f(x)dx
\end{aligned}$$

Using the fact that

$$\int_{-\infty}^{\infty} x^2 f(x)dx = \mu_2', \quad \int_{-\infty}^{\infty} x f(x)dx = \mu \quad \text{and} \quad \int_{-\infty}^{\infty} f(x)dx = 1$$

we have

$$\sigma^2 = \mu_2' - 2\mu^2 + \mu^2 = \mu_2' - \mu^2$$

5.13 The density is

$$f(x) = \begin{cases} 4x^3 & 0 < x < 1 \\ 0 & \text{elsewhere} \end{cases}$$

Thus,

$$\mu = \int_0^1 4x^4 dx = 4x^5/5\Big|_0^1 = 4/5$$

$$\mu_2' = \int_0^1 4x^5 dx = 4x^6/6\Big|_0^1 = 2/$$

and the variance is

$$\sigma^2 = \mu_2' - \mu^2 = 2/3 - (4/5)^2 = .0267$$

5.14 In this case,

$$\begin{aligned}\mu &= \int_0^2 xf(x)dx = \int_0^1 x^2dx + \int_1^2 x(2-x)dx \\ &= x^3/3\Big|_0^1 + x^2\Big|_1^2 - x^3/3\Big|_1^2 = 1/3+4-1-8/3+1/3 = 1\end{aligned}$$

and

$$\begin{aligned}\mu_2' &= \int_0^2 x^2 f(x)dx = \int_0^1 x^3dx + \int_1^2 x^2(2-x)dx \\ &= x^4/4\Big|_0^1 + 2x^3/3\Big|_1^2 - x^4/4\Big|_1^2 \\ &= 1/4 + 16/3 - 2/3 - 16/4 + 1/4 = 7/6\end{aligned}$$

Thus,

$$\sigma^2 = \mu_2' - \mu^2 = 7/6 - 1^2 = 1/6$$

5.15 The density is:

$$f(x) = \begin{cases} 8x^{-3} & x > 2 \\ 0 & x \leq 2 \end{cases}$$

Thus,

$$\mu = \int_2^\infty x(8x^{-3})dx = -8x^{-1}\Big|_2^\infty = 4$$

and

$$\mu_2' = \int_2^\infty x^2(8x^{-3})dx = 8\ln x|_2^\infty = \infty$$

Thus, σ^2 does not exist.

5.16 The density is

$$f(x) = \begin{cases} \cos x & 0 < x < \pi/2 \\ 0 & \text{elsewhere} \end{cases}$$

Thus,

$$\begin{aligned}\mu &= \int_0^{\pi/2} x\cos x dx = x\sin x\Big|_0^{\pi/2} - \int_0^{\pi/2} \sin x dx \\ &= \pi/2 + \cos x\Big|_0^{\pi/2} = \pi/2 - 1\end{aligned}$$

and

$$\begin{aligned}\mu_2' &= \int_0^{\pi/2} x^2 \cos x dx = x^2 \sin x\Big|_0^{\pi/2} - 2\int_0^{\pi/2} x\sin x dx \\ &= x^2 \sin x\Big|_0^{\pi/2} - 2\left[- x\cos x\Big|_0^{\pi/2} + \int_0^{\pi/2} \cos x dx\right] \\ &= \pi^2/4 - 2[1-0] = \pi^2/4 - 2\end{aligned}$$

Thus,

$$\sigma^2 = \pi^2/4 - 2 - \pi^2/4 + 2\pi/2 - 1 = \pi - 3$$

5.17 The density is:

$$f(x) = \begin{cases} (1/20)e^{-x/20} & x > 0 \\ 0 & x \le 0 \end{cases}$$

Thus,

$$\mu = \frac{1}{20}\int_0^\infty xe^{-x/20}dx$$

Integrating by parts gives:

$$\begin{aligned}\mu &= -xe^{-x/20}\Big|_0^\infty + \int_0^\infty e^{-x/20}dx \\ &= 0 - 20e^{-x/20}\Big|_0^\infty \\ &= 20 \text{ (thousand miles)}\end{aligned}$$

5.18 On $|x| > 1$, we have $x^2/(1+x^2) > 1$. So

$$\mu_2' = \frac{1}{\pi}\int_{-\infty}^{\infty} \frac{x^2}{1+x^2}dx = \frac{2}{\pi}\int_{0}^{\infty} \frac{x^2}{1+x^2}dx > \frac{2}{\pi}\int_{1}^{\infty} dx$$

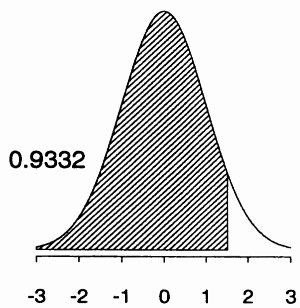
and this integration term is infinite. Thus μ_2' is infinite and the variance does not exist.

5.19 (a) $P(\text{less than } 1.50) = F(1.50) = .9332$

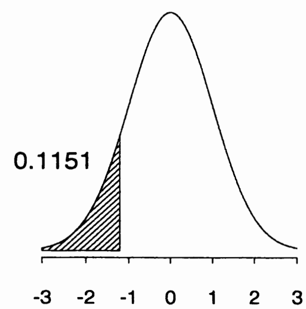
(b) $P(\text{less than } -1.20) = F(-1.20) = .1151$

(c) $P(\text{greater than } 2.16) = 1 - F(2.16) = 1 - .9846 = .0154$

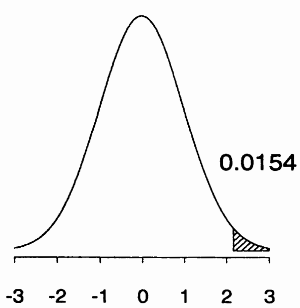
(d) $P(\text{greater than } -1.75) = F(1.75) = .9599$



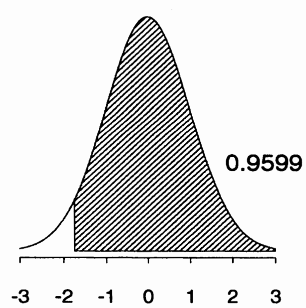
5.19 (a) z



5.19 (b) z



5.19 (c) z



5.19 (d) z

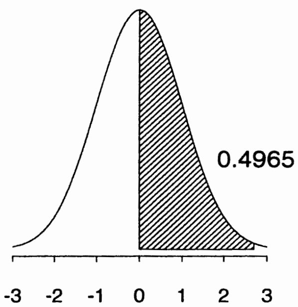
5.20 Let Z be a random variable having a standard normal distribution.

(a) $P(0 < Z < 2.7) = F(2.7) - F(0) = .9965 - .50 = .4965$

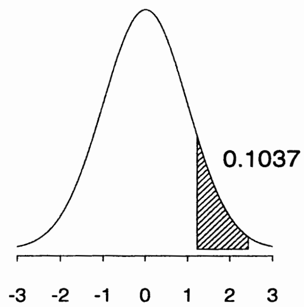
(b) $P(1.22 < Z < 2.43) = F(2.43) - F(1.22) = .9925 - .8888 = .1037$

(c) $P(-1.35 < Z < -.35) = F(-.35) - F(-1.35) = .3632 - .0885 = .2747$

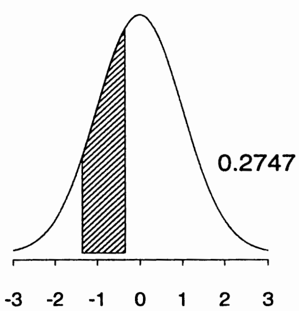
(d) $P(-1.70 < Z < 1.35) = F(1.35) - F(-1.70) = .9115 - .0446 = .8669$



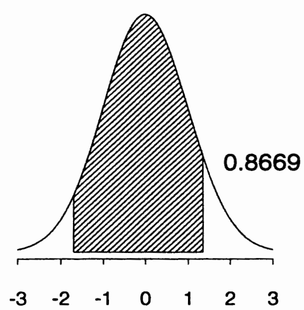
5.20 (a) z



5.20 (b) z



5.20 (c) z



5.20 (d) z

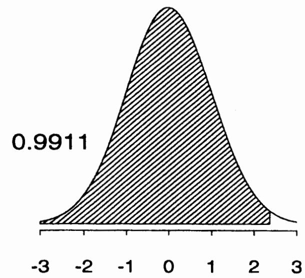
5.21 (a) $P(Z \le z) = F(z) = .9911$. Thus $z = 2.37$

(b) $P(Z > z) = .1093$. That is, $P(Z \le z) = 1 - .1093$ or $F(z) = .8907$. Thus, $z = 1.23$

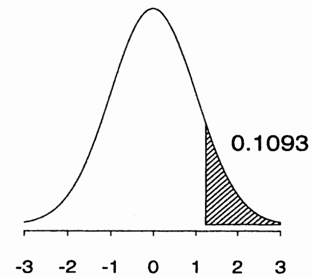
(c) $P(Z > z) = .6443$. That is, $F(z) = 1 - .6443 = .3557$. Using Table 3, $z = -.37$

(d) $P(Z < z) = .0217$ so z is negative. From Table 3, $z = -2.02$.

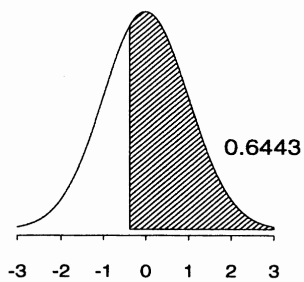
(e) $P(-z \le Z \le z) = .9298$. That is, $F(z) - F(-z) = .9298$, which implies that $F(z) - (1 - F(z)) = .9298$ or $F(z) = (1 + .9298)/2 = .9649$. By Table 3, $z = 1.81$.



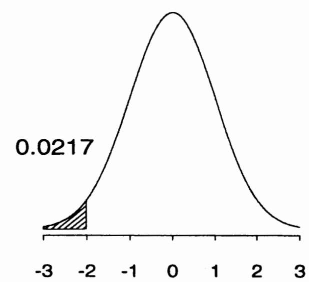
5.21 (a) z



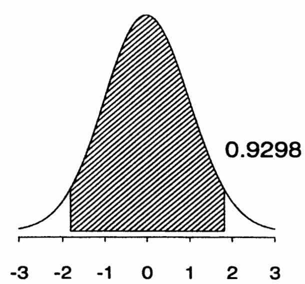
5.21 (b) z



5.21 (c) z



5.21 (d) z



5.21 (e) z

5.22 Let X be a random variable having distribution $N(\mu, \sigma^2)$ (normal with mean μ and variance σ^2). Then

(a) $P(|X-\mu| < \sigma) = P(|X-\mu|/\sigma < 1) = P(-1 < (X-\mu)/\sigma < 1)$
$= F(1) - 1 + F(1) = 2F(1) - 1 = .6826$

(b) $P(|X-\mu| < 2\sigma) = 2F(2) - 1 = .9544$

(c) $P(|X-\mu| < 3\sigma) = 2F(3) - 1 = .9974$

(d) $P(|X-\mu| < 4\sigma) = 2F(4) - 1 = 2(.99997) - 1 = .99994$

5.23 (a) $P(Z > z_{.005}) = .005$. Thus, $F(z_{.005}) = .995$ and $z = 2.575$ by linear interpolation in the Table 3.

(b) $P(Z > z_{.025}) = .025$. Thus, $F(z_{.025}) = .975$ and $z = 1.96$

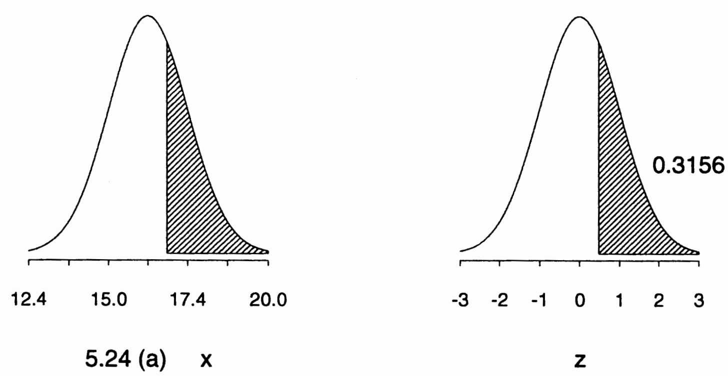
5.24 Let X have distribution $N(16.2, 1.5625)$.

(a) $P(X > 16.8) = 1 - F((16.8 - 16.2)/1.25) = 1 - F(.48) = 1 - .6844 = .3156$

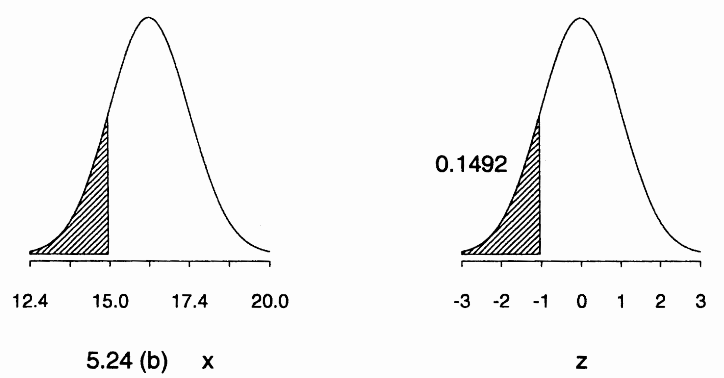
(b) $P(X < 14.9) = F((14.9 - 16.2)/1.25) = F(-1.04) = .1492$

(c) $P(13.6 < X < 18.8) = F((18.8 - 16.2)/1.25) - F((13.6 - 16.2)/1.25)$
$= F(2.08) - F(-2.08) = .9812 - .0188 = .9624$

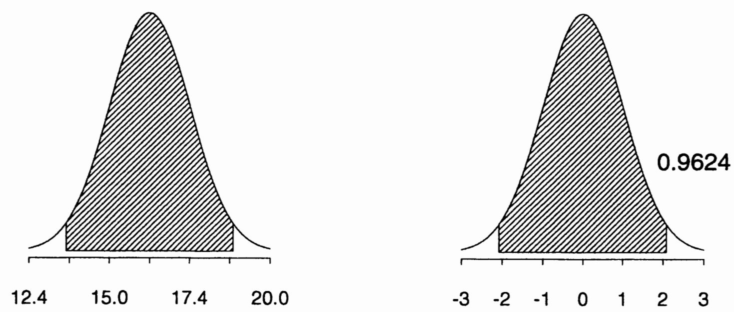
(d) $P(16.5 < X < 16.7) = F((16.7 - 16.2)/1.25) - F((16.5 - 16.2)/1.25)$
$= F(.4) - F(.24) = .6554 - .5948 = .0606$



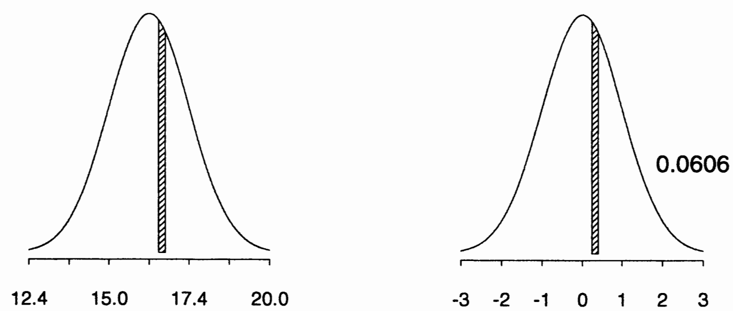
5.24 (a)



5.24 (b)



5.24 (c) x z



5.24 (d) x z

5.25

$$P[X > 39.2] = .20 \quad \text{so} \quad P[\frac{X-30}{\sigma} > \frac{9.2}{\sigma}] = .20$$

That is, $1 - F(9.2/\sigma) = .20$, and $F(9.2/\sigma) = .80$. But $F(.842) = .80$. Thus $9.2/\sigma = .842$, so $\sigma = 10.93$.

5.26 From Table 3, $P((X-\mu)/10 < .92) = .8212$. Thus, given

$$P[X < 282.5] = .8212 \quad \text{or} \quad P[\frac{X-\mu}{10} > \frac{282.5-\mu}{10}] = .8212$$

we maut have $(282.5 - \mu)/10 = .92$ or $\mu = 282.5 - 9.2 = 273.3$ To find $P(X > 258.3)$, we need to find

$$1 - F((258.3 - 273.3)/10) = 1 - F(-15/10) = F(1.5)$$

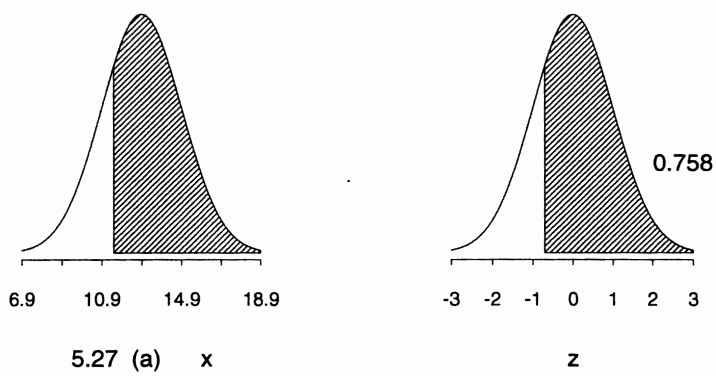
From Table 3, $F(1.5) = .9332$. Thus, $P(X > 258.3) = .9332$

5.27 (a) We need to find $P(X > 11.5)$, where X is normally distributed with $\mu = 12.9$ and $\sigma = 2$.

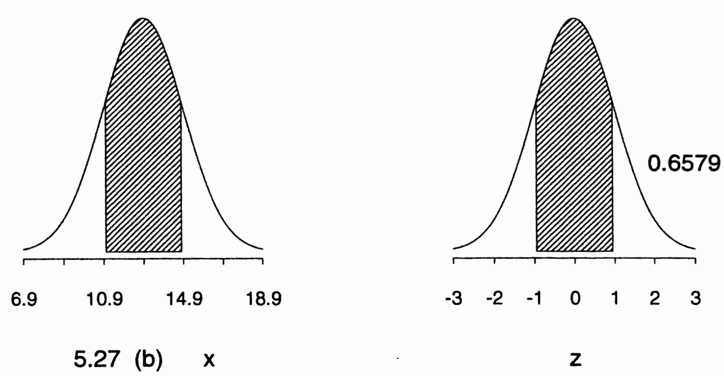
$$P(X \geq 11.5) = 1 - F((11.5 - 12.9)/2) = F((12.9 - 11.5)/2) = F(.7) = .7580$$

(b)

$$P(11 \leq X \leq 14.8) = F((14.8 - 12.9)/2) - F((11 - 12.9)/2) = F(.95) - F(-.95) = .8289 - .1711 = .6578$$



5.27 (a)



5.27 (b)

5.28 $P(Z \leq -z_{0.25}) = .25$. Thus, $-z_{0.25} = -.675$

$P(Z \leq z_{0.50}) = F(z_{0.50}) = .50$. Thus, $z_{0.50} = 0$

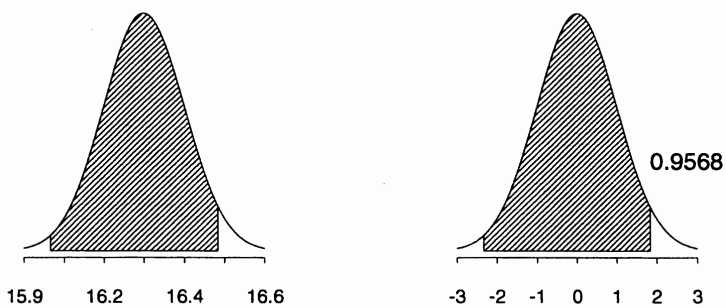
$P(Z \leq z_{0.25}) = F(z_{0.25}) = .75$. Thus, $z_{0.25} = .675$

5.29 Let X be a random variable representing the developing time which is normally distributed with $\mu = 16.28$ and $\sigma = .12$.

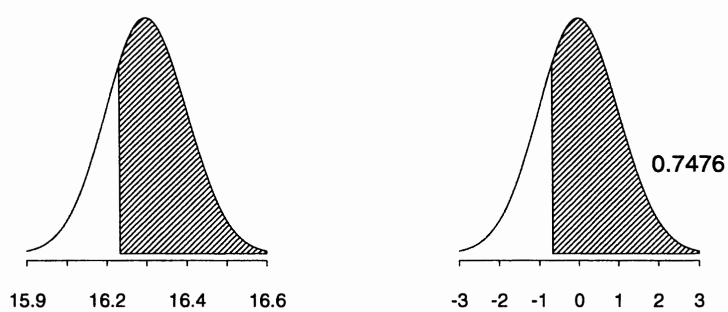
(a) $P(16 \leq X \leq 16.5) = F((16.5 - 16.28)/.12) - F((16 - 16.28)/.12)$
$= F(1.833) - F(-2.333) = .9666 - .0098 = .9568$
(These values are determined by interpolation)

(b) $P(X \geq 16.20) = 1 - F((16.20 - 16.28)/.12) = 1 - F(-.667)$
$= F(.667) = .7476$

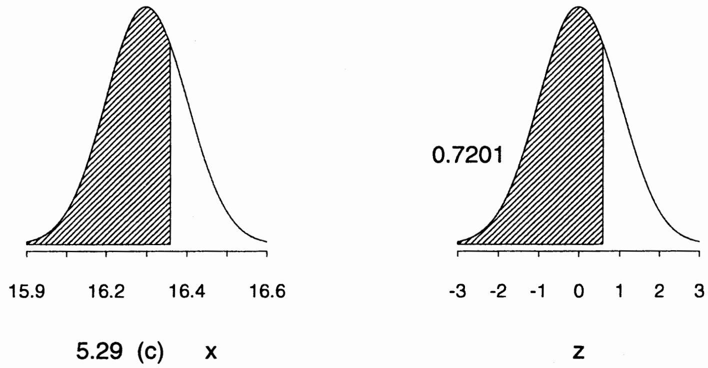
(c) $P(X \leq 16.35) = F((16.35 - 16.28)/.12) = F(.5833) = .7201$



5.29 (a) x z



5.29 (b) x z

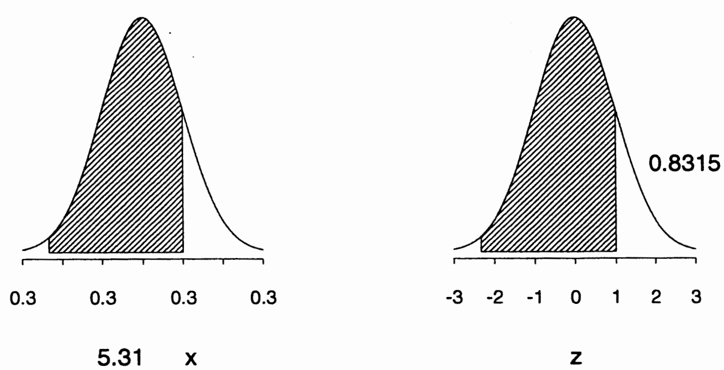


5.30 From Table 3, we know that $F(1.645) = .95$. We need to find a value x such that

$$\begin{aligned} P(X > x) &= P((X - 16.28)/.12 > (x - 16.28)/.12) \\ &= 1 - P((X - 16.28)/.12 < (x - 16.28)/.12) \\ &= 1 - F((x - 16.28)/.12\ ) \ = \ F((\ 16.28 - x)/.12) \ = \ .95 \end{aligned}$$

Thus, $(16.28 - x)/.12 = 1.645$, and $x = 16.0826$

5.31 $P(.295 \leq X \leq .305) = F((.305 - .302)/.003) - F((.295 - .302)/.003)$
$= F(1) - F(-2.333) = .8413 - .0098 = .8315$
Thus, 83.15 percent will meet specifications.



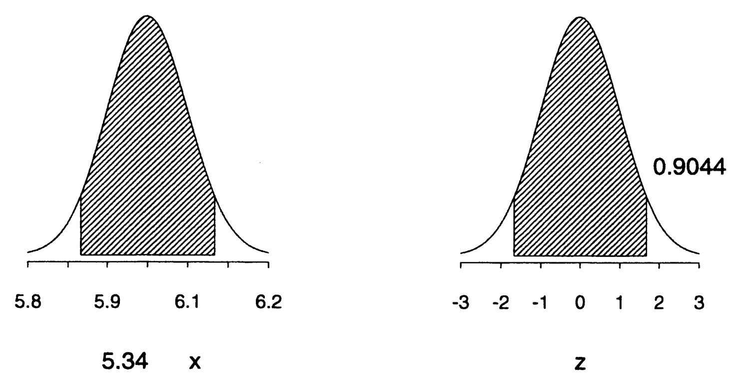
5.32 We must find μ such that $F((4 - \mu)/.025) = .02$ or $F((\mu - 4)/.025) = .98$. But, $F(2.05) = .98$. Thus, $(\mu - 4)/.025 = 2.05$ or $\mu = 4.05$.

5.33 We need to find μ such that $F((3-\mu)/.01) = .95$. Thus, from Table 3, $(3-\mu)/.01 = 1.645$ or $\mu = 2.98355$.

5.34 (a)

$$\begin{aligned} P(5.9 \le X \le 6.1) &= F((6.1-6)/.06) - F((5.9-6)/.06) \\ &= F(1.667) - F(-1.667) = .9522 - .0478 = .9044 \end{aligned}$$

The proportion of rods exceeding the tolerance limits is $1 - .9044 = .0956$



5.34

(b) If 99% of the rods must be within tolerance, σ should satisfy $2F(.1/\sigma) - 1 = .99$, that is $F(.1/\sigma) = .995$. Hence $.1/\sigma = 2.575$ or $\sigma = 0.0388$

5.35 If $n = 30$ and $p = .60$ then $\mu = 30(.60) = 18$ and $\sigma^2 = 30(.6)(.4) = 7.2$ or $\sigma = 2.6833$.

(a) $P(14) = F((14.5-18)/2.6833) - F((13.5-18)/2.6833)$
$= F(-1.304) - F(-1.677) = .0961 - .0468 = .0493$

(b) $P(\text{less than } 12) = F((11.5-18)/2.6833) = F(-2.42) = .0078$

5.36 In this case, $n = 1200$, $p = .02$, $\mu = 24$, $\sigma^2 = 23.52$, $\sigma = 4.8497$. Thus,

$$\begin{aligned} P(\text{at least 30 need repairs}) &= 1 - F((29.5-24)/4.8497) \\ &= 1 - F(1.13) = 1 - .8707 = .1292 \end{aligned}$$

5.37 In this case, $n = 200$, $p = .25$, $\mu = 50$, $\sigma^2 = 37.5$, $\sigma = 6.1237$. Thus,

$$\begin{aligned} P(\text{fewer than 45 fail}) &= F((44.5 - 50)/6.1237) \\ &= F(-.90) \;=\; .1841 \end{aligned}$$

5.38 In this case, $n = 84$, $p = .3$, $\mu = 25.2$, $\sigma^2 = 17.64$, $\sigma = 4.2$.

$$\begin{aligned} & F((30.5 - 25.2)/4.2) - F((19.5 - 25.2)/4.2) \\ &= F(1.26) - F(-1.36) \\ &= .8962 - .0869 \;=\; .8093 \end{aligned}$$

5.39 Again, we will use the normal approximation to the binomial distribution. Here, $n = 40$, $p = .62$, $\mu = 24.8$, $\sigma^2 = 9.424$, $\sigma = 3.0699$. Thus,

$$F((20.5 - 24.8)/3.0699) = F(-1.40) \;=\; .0808$$

5.40 (a) We will use the normal approximation to the binomial with $n = 1,000$, $p = .5$, $\mu = 500$, $\sigma^2 = 250$, $\sigma = 15.81$. The proportion from .49 to .51 means the actual number is from 490 to 510. Thus,

$$\begin{aligned} & F((510.5 - 500)/15.81) - F((489.5 - 500)/15.81) \\ &= F(.664) - F(-.664) - \;=\; .7467 - .2533 \;=\; .4934 \end{aligned}$$

(b) Similar to part (a), with parameters $n = 10,000$, $p = .5$, $\mu = 5,000$, $\sigma^2 = 2500$, $\sigma = 50$. Thus,

$$\begin{aligned} & F((5100.5 - 5000)/50) - F((4899.5 - 5000)/50) \\ &= F(2.01) - F(-2.01) \;=\; .9778 - .0222 \;=\; .9556 \end{aligned}$$

5.41 Let $f(x)$ be the standard normal density. Then $F(-z) = \int_{-\infty}^{-z} f(x)dx$. Using the change of variable, $s = -x$, and the fact that $f(x) = f(-x)$, we have

$$F(-z) = -\int_{\infty}^{z} f(-s)ds = \int_{z}^{\infty} f(s)ds = 1 - \int_{-\infty}^{z} f(s)ds = 1 - F(z)$$

5.42 To find the mean of the normal density, we need to find

$$\begin{aligned}
&\left(2\pi\sigma^2\right)^{-1/2} \int_{-\infty}^{\infty} x \exp\left[-\frac{(x-\mu)^2}{2\sigma^2}\right] dx \\
= \;& (2\pi)^{-1/2} \int_{-\infty}^{\infty} \left[\frac{x-\mu}{\sigma} + \frac{\mu}{\sigma}\right] \exp\left[-\frac{1}{2}\left(\frac{x-\mu}{\sigma}\right)^2\right] dx \\
= \;& (2\pi)^{-1/2} \int_{-\infty}^{\infty} \left[\frac{x-\mu}{\sigma}\right] \exp\left[-\frac{1}{2}\left(\frac{x-\mu}{\sigma}\right)^2\right] dx + \mu
\end{aligned}$$

Using the change of variable $u = (x-\mu)/\sigma$ yields an odd integrand, $u\exp(-u^2/2)$ so the interal is 0. Thus, the mean is μ.

5.43 We need to find

$$\begin{aligned}
&\frac{1}{\sqrt{2\pi\sigma^2}} \int_{-\infty}^{\infty} (x-\mu)^2 \exp\left[-\frac{(x-\mu)^2}{2\sigma^2}\right] dx \\
= \;& \frac{2}{\sqrt{2\pi\sigma^2}} \int_{\mu}^{\infty} (x-\mu)^2 \exp\left[-\frac{(x-\mu)^2}{2\sigma^2}\right] dx
\end{aligned}$$

since the integrand is an even function. Using the change of variable $s = (x-\mu)/\sigma$, the variance is equal to

$$\frac{2\sigma^2}{\sqrt{2\pi}} \int_{0}^{\infty} s^2 \exp[-s^2/2]ds$$

Integrating by parts with $u = s$ and $dv = s\exp[-s^2/2]/(2\pi)^{1/2}ds$ shows that the variance is equal to

$$2\sigma^2 \left[-s \cdot \exp(-s^2/2)/(2\pi)^{1/2} \,|_0^\infty + \int_0^\infty \exp[-s^2/2]/(2\pi)^{1/2} ds\right]$$

The first term is zero and the second is 1/2 since this is an integration of half of the standard normal density. Thus the variance is σ^2.

5.44 (a) `Normal with mean = 11.3000 and standard deviation = 5.70000`

x	P(X <= x)
8.4930	0.3112

(b) `Normal with mean = 11.3000 and standard deviation = 5.70000`

x	P(X <= x)
16.0740	0.7989

5.45 The uniform density is:

$$f(x) = \begin{cases} 1/(\beta-\alpha) & \alpha < x < \beta \\ 0 & \text{elsewhere} \end{cases}$$

Thus, the distribution function is

$$F(x) = \begin{cases} 1 & x \geq \beta \\ (x-\alpha)/(\beta-\alpha) & \alpha < x < \beta \\ 0 & x \leq \alpha \end{cases}$$

5.46 (a) $P(.010 \leq \text{ error } \leq .015) = (.015 - .010)/.050 = .1$

(b) $P(-.012 \leq \text{ error } \leq .012) = (.012 + .012)/.050 = .48$

5.47 Suppose Mr. Harris bids $(1+x)c$. Then his expected profit is:

$$\begin{aligned} &0P(\text{low bid } < (1+x)c) + xcP(\text{low bid } \geq (1+x)c) \\ &= xc\int_{(1+x)c}^{2c} \frac{3}{4c} ds = 3xc[2c-(1+x)c]/4c = 3c(x-x^2)/4 \end{aligned}$$

Thus, his profit is maximum when $x = 1/2$. So his bid is 3/2 times his cost. Thus, he adds 50 percent to his cost estimate.

5.48

$$\frac{1}{\sqrt{2\pi}\beta}\int_0^\infty x x^{-1}\exp[-(\ln x-\alpha)^2/(2\beta^2)]dx$$

$$=\frac{1}{\sqrt{2\pi}\beta}\int_0^\infty \exp[-(\ln x-\alpha)^2/(2\beta^2)]dx$$

Using the change of variable $y=\ln x$ gives:

$$\begin{aligned}
\mu &= \frac{1}{\sqrt{2\pi}\beta}\int_{-\infty}^\infty e^y\exp[-(y-\alpha)^2/(2\beta^2)]dy \\
&= \frac{1}{\sqrt{2\pi}\beta}\int_{-\infty}^\infty \exp[y-(y-\alpha)^2/(2\beta^2)]dy \\
&= \frac{1}{\sqrt{2\pi}\beta}\int_{-\infty}^\infty \exp[-(y^2-2y\alpha+\alpha^2-2\beta^2 y)/(2\beta^2)]dy \\
&= \frac{1}{\sqrt{2\pi}\beta}\int_{-\infty}^\infty \exp[-(y^2-2(\alpha+\beta^2)y+\alpha^2)/(2\beta^2)]dy \\
&= \frac{1}{\sqrt{2\pi}\beta}\int_{-\infty}^\infty \exp[-((y-(\alpha+\beta^2))^2-(\alpha+\beta^2)^2+\alpha^2)/(2\beta^2)]dy \\
&= \frac{1}{\sqrt{2\pi}\beta}\int_{-\infty}^\infty \exp[-((y-(\alpha+\beta^2))^2-2\alpha\beta^2-\beta^4)/(2\beta^2)]dy \\
&= \exp[\alpha+\beta^2/2]\cdot\frac{1}{\sqrt{2\pi}\beta}\int_{-\infty}^\infty \exp[-(y-(\alpha+\beta^2))^2/(2\beta^2)]dy \\
&= \exp[\alpha+\beta^2/2]
\end{aligned}$$

5.49 I_0/I_i is distributed log-normal with $\alpha=2$, $\beta^2=.01$, $\beta=.1$. Thus,

$$\begin{aligned}
P(7\le I_0/I_i\le 7.5) &= F((\ln(7.5)-2)/.1)-F((\ln(7)-2)/.1) \\
&= F(.149)-F(-.54) \\
&= .5592-.2946=.2646
\end{aligned}$$

5.50 Using the formulas for μ and σ^2 for the log-normal distribution with $\alpha=-1$ and

$\beta = 2$, gives:

$$\begin{aligned}
\mu &= \exp[\alpha + \beta^2/2] = \exp[-1 + 4/2] = e^1 = e = 2.718 \\
\sigma^2 &= e^{2\alpha+\beta^2}[e^{\beta^2} - 1] = e^{-2+4}[e^4 - 1] = e^2[e^4 - 1] = e^6 - e^2 = 396
\end{aligned}$$

Thus, $\sigma = 19.9$

5.51 (a) $P(\text{between 3.2 and 8.4}) = F((\ln(8.4) + 1)/2) - F((\ln(3.2) + 1)/2)$
$= F(1.564) - F(1.0816) = .9406 - .8599 = .0807$

(b) $P(\text{greater than 5}) = 1 - F((\ln(5) + 1)/2) = 1 - F(1.305) = 1 - .904 = .0960$

5.52 Using the formulas for μ and σ^2 for the gamma distribution with $\alpha = 2$ and $\beta = 2$, gives $\mu = \alpha\beta = 4$ and, $\sigma^2 = \alpha\beta^2 = 8$. Thus, $\sigma = 2.8284$.

5.53 When $\alpha = 2$ and $\beta = 2$, $\Gamma(2) = 1$. So

$$f(x) = \begin{cases} xe^{-x/2}/4 & x > 0 \\ 0 & x \le 0 \end{cases}$$

Thus,

$$P(X < 4) = \int_0^4 f(x)dx = \frac{1}{4}\int_0^4 xe^{-x/2}dx$$

Integrating by parts gives

$$\begin{aligned}
-\frac{1}{2}xe^{-x/2}\Big|_0^4 + \frac{1}{2}\int_0^4 e^{-x/2}dx &= 2e^{-2} - e^{-x/2}\Big|_0^4 \\
&= 1 - 3e^{-2} = .5940
\end{aligned}$$

5.54 The density of the gamma distribution with $\alpha = 3$, $\beta = 2$ is

$$f(x) = \begin{cases} x^2e^{-x/2}/16 & x > 0 \\ 0 & x \le 0 \end{cases}$$

Thus,

$$P(\text{power inadequate}) = \frac{1}{16}\int_{12}^{\infty} x^2 e^{-x/2} dx$$

Integrating by parts gives

$$(1/16)\left[-2x^2 e^{-x/2}\,|_{12}^{\infty} + 4\int_{12}^{\infty} x e^{-x/2} dx\right]$$

Integrating the second term by parts gives

$$(1/16)\left[-2x^2 e^{-x/2}\,|_{12}^{\infty} + 4\left(-2xe^{-x/2}\,|_{12}^{\infty} + 2\int_{12}^{\infty} e^{-x/2} dx\right)\right]$$

$$= (1/16)(-2x^2 e^{-x/2} - 8xe^{-x/2} - 16e^{-x/2})\,|_{12}^{\infty} = 25e^{-6} = .062$$

5.55 (a) The probability that the supports will survive, if $\mu = 3.0$ and $\sigma^2 = .09$, is

$$\begin{aligned} P(\text{supports will survive}) &= 1 - F\left(\frac{\ln(33) - 3.0}{.30}\right) = 1 - F(1.655) \\ &= 1 - .9508 = .0492 \end{aligned}$$

(b) If $\mu = 4.0$ and $\sigma^2 = .36$, then

$$\begin{aligned} P(\text{supports will survive}) &= 1 - F\left(\frac{\ln(33) - 4.0}{.60}\right) = 1 - F(-.84) \\ &= F(.84) = .7995 \end{aligned}$$

5.56

$$\mu_2' = \frac{1}{\beta^\alpha \Gamma(\alpha)}\int_0^{\infty} x^2 x^{\alpha-1} e^{-x/\beta} dx = \frac{1}{\beta^\alpha \Gamma(\alpha)}\int_0^{\infty} x^{\alpha+1} e^{-x/\beta} dx$$

Using the change of variable $y = x/\beta$ gives

$$\mu_2' = \frac{1}{\beta^\alpha \Gamma(\alpha)}\int_0^{\infty} (\beta y)^{\alpha+1} e^{-y} \beta dy = \frac{\beta^2}{\Gamma(\alpha)}\int_0^{\infty} y^{\alpha+1} e^{-y} dy$$

$$= \beta^2 \frac{\Gamma(\alpha+2)}{\Gamma(\alpha)} = \beta^2(\alpha+1)\alpha$$

Thus,

$$\sigma^2 = \beta^2(\alpha+1)\alpha - \alpha^2\beta^2 = \alpha\beta^2$$

5.57 We can ignore the constant in the density since it is always positive. Thus, we need to maximize $f(x) = x^{\alpha-1}e^{-x/\beta}$. Taking the derivative

$$f'(x) = (\alpha-1)x^{\alpha-2}e^{-x/\beta} - x^{\alpha-1}e^{-x/\beta}/\beta = x^{\alpha-2}e^{-x/\beta}(\alpha-1-x/\beta)$$

Setting the derivative equal to zero gives the solution $x = \beta(\alpha-1)$. For $\alpha > 1$, the derivative is positive for $x < \beta(\alpha-1)$ and negative for $x > \beta(\alpha-1)$. Thus, $\beta(\alpha-1)$ is a maximum. Note that $x = 0$ is a point of inflection when $\alpha > 2$. When $\alpha = 1$, $f(x) = e^{-x/\beta}$ which has a maximum in the interval $[0,\infty]$ at $x = 0$. When $0 < \alpha < 1$, the derivative does not vanish on $(0,\infty)$ and $f(x)$ is unbounded as x decreases to 0.

5.58 The exponential density is

$$f(x) = \begin{cases} e^{-x/\beta}/\beta & x > 0, \beta > 0 \\ 0 & \text{elsewhere} \end{cases}$$

and the exponential distribution is

$$F(x) = \int_0^x f(s)ds = 1 - e^{-x/\beta} \quad \text{for} \quad x > 0.$$

(a) $P(X \leq 20) = 1 - e^{-20/50} = 1 - e^{-2/5} = .3297$

(b) $P(X \geq 60) = 1 - (1 - e^{-60/50}) = e^{-6/5} = .3012$

5.59 Since the number of breakdowns is a Poisson random variable with parameter $\lambda = .3$, the interval between breakdowns is an exponential random variable with parameter $\lambda = .3$.

(a) The probability that the interval is less than 1 week is $1 - e^{-(.3)1} = .259$ or 25.9 percent.

(b) The probability that the interval is greater than 5 weeks is $e^{-(.3)5} = .223$ or 22.3 percent.

5.60 Assuming that the time between calls is distributed as an exponential random variable with $\lambda = .6$, the probability that there are no arrivals in an interval of length t is $e^{-\lambda t} = e^{-.6t}$.

5.61 Let N be a random variable having the Poisson distribution with parameter αt. Then $P(N = 0) = (\alpha t)^0 e^{-\alpha t}/0! = e^{-\alpha t}$. Thus, $P(\text{waiting time is} > t) = e^{-\alpha t}$ and $P(\text{waiting time is } \leq t) = 1 - e^{-\alpha t}$.

5.62 The density is given by the derivative of P (waiting time is $\leq t$). Thus, the density is $\alpha e^{-\alpha t}$.

5.63 The beta density is

$$f(x) = \frac{\Gamma(\alpha+\beta)}{\Gamma(\alpha)\Gamma(\beta)} x^{\alpha-1}(1-x)^{\beta-1}$$

for $0 < x < 1$, $\alpha > 0$, and $\beta > 0$. For $\alpha = 3$ and $\beta = 3$

$$f(x) = \frac{\Gamma(6)}{\Gamma(3)\Gamma(3)} x^2(1-x)^2 = \frac{5!}{2!2!} x^2(1-x)^2 = 30(x^2 - 2x^3 + x^4)$$

Thus,

$$\begin{aligned} \int_0^1 f(x)dx &= 30\int_0^1 (x^2 - 2x^3 + x^4)dx = 30(x^3/3 - x^4/2 + x^5/5)\,\Big|_0^1 \\ &= 30(1/3 - 1/2 + 1/5) = 1 \end{aligned}$$

as required.

5.64 When $\alpha = 2$ and $\beta = 9$, the beta density is

$$\frac{\Gamma(11)}{\Gamma(2)\Gamma(9)}x(1-x)^8 = \frac{10!}{1!8!}x(1-x)^8 = 90x(1-x)^8$$

Thus, the required probability is given by

$$\begin{aligned} 90\int_0^{.1} x(1-x)^8 dx &= 90\int_0^{.1}(x-1+1)(1-x)^8 dx \\ &= 90[(1-x)^{10}/10 - (1-x)^9/9)\ |_0^{.1}] \\ &= 90[(.9^{10}-1)/10 - (.9^9-1)/9] = .2639 \end{aligned}$$

5.65 (a) The mean of the beta distribution is given by $\mu = \alpha/(\alpha+\beta)$. Thus, in the case where $\alpha = 1$ and $\beta = 4$, $\mu = 1/(1+4) = 1/5 = .2$

(b) When $\alpha = 1$ and $\beta = 4$, the beta density is

$$\frac{\Gamma(5)}{\Gamma(1)\Gamma(4)}x^0(1-x)^3 = \frac{4!}{0!3!}(1-x)^3 = 4(1-x)^3$$

Thus, the required probability is given by

$$4\int_{.25}^{1}(1-x)^3 dx = -(1-x)^4\ |_{.25}^{1} = (.75)^4 = .3164$$

5.66 Since the coefficient $\Gamma(\alpha+\beta)/\Gamma(\alpha)\Gamma(\beta)$ is always > 0, we need to find the maximum of $f(x) = x^{\alpha-1}(1-x)^{\beta-1}$. When $\alpha > 1$, $\beta > 1$, taking the derivative gives

$$\begin{aligned} f'(x) &= x^{\alpha-2}(1-x)^{\beta-2}((\alpha-1)(1-x) - (\beta-1)x) \\ &= x^{\alpha-2}(1-x)^{\beta-2}((\alpha-1) - (\alpha+\beta-2)x) \end{aligned}$$

Thus, the derivative is 0 when $x = (\alpha-1)/(\alpha+\beta-2)$. It also equals 0 at $x = 0$ and $x = 1$ if $\alpha > 2$ and $\beta > 2$ respectively. Since $f(0) = 0$, $f(1) = 0$, $f(x) \geq 0$, and $f'(x)$ is continuous, $x = (\alpha-1)/(\alpha+\beta-2)$ is a maximum.

5.67 Let X be Weibull random variable with $\alpha = .1$, $\beta = .5$ representing the battery lifetime. Then the density is $f(x) = (.1)(.5)x^{-.5}e^{-.1x^{.5}}$ for $x > 0$. Thus,

$$P(X \leq 100) = \int_0^{100} (.1)(.5)x^{-.5}e^{-.1x^{.5}}dx$$

Using the change of variable $y = x^{.5}$ gives:

$$P(X \leq 100) = .1\int_0^{10} e^{-.1y}dy = -e^{-.1y}\,|_0^{10} = 1 - e^{-1} = .6321$$

5.68 (a) The expected value of the Weibull random variable is given by

$$\mu = \alpha^{-1/\beta}\Gamma(1 + 1/\beta)$$

When $\alpha = 1/5$ and $\beta = 1/3$,

$$\mu = (1/5)^{-3}\Gamma(4) = 125 \cdot 6 = 750$$

(b) The probability is given by

$$\frac{1}{5}\cdot\frac{1}{3}\int_0^{300} x^{1/3-1}\exp(\frac{-x^{1/3}}{5})dx = \int_0^{300^{1/3}} \frac{1}{5}e^{-y/5}dy$$

$$= 1 - \exp(\frac{-300^{1/3}}{5}) = 1 - e^{-1.3389} = .7379$$

5.69 The probability is

$$\begin{aligned}\int_{4,000}^{\infty} (.025)(.500)x^{-.5}e^{-(.025)x^{.500}}dx &= \int_{\sqrt{4,000}}^{\infty} .025e^{-.025y}dy \\ &= e^{-.025\sqrt{4,000}} = .2057\end{aligned}$$

5.70

$$\mu_2' = \int_0^\infty x^2 \alpha\beta x^{\beta-1} e^{-\alpha x^\beta} dx = \alpha\beta \int_0^\infty x^{\beta+1} e^{-\alpha x^\beta} dx$$

Let $y = \alpha x^\beta$. Then $dy = \alpha\beta x^{\beta-1} dx$. Thus,

$$\mu_2' = \int_0^\infty \left(\frac{y}{\alpha}\right)^{2/\beta} e^{-y} dy = \alpha^{-2/\beta} \int_0^\infty y^{2/\beta} e^{-y} dy = \alpha^{-2/\beta}\Gamma(\frac{2}{\beta}+1)$$

Thus, $\sigma^2 = \alpha^{-2/\beta}\left[\Gamma(1+2/\beta) - (\Gamma(1+1/\beta))^2\right]$

5.71 (a) The joint probability distribution of X_1 and X_2 is

$$f(x_1, x_2) = \frac{\binom{2}{x_1}\binom{1}{x_2}\binom{2}{2-x_1-x_2}}{\binom{5}{2}},$$

where $x_1 = 0, 1, 2$, $x_2 = 0, 1$, and $0 \le x_1 + x_2 \le 2$. The joint probability distribution $f(x_1, x_2)$ can be summarized in the following table:

		X_2		
	$f(x_1, x_2)$	0	1	Total
	0	.1	.2	.3
X_1	1	.4	.2	.6
	2	.1	0	.1
	Total	.6	.4	1

(b) Let A be the event that $X_1 + X_2 = 0$ or 1, then

$$P(A) = f(0,0) + f(0,1) + f(1,0) = .1 + .2 + .4 = .7$$

(c) By (a), the marginal distribution of X_1 is

$$\begin{aligned} f_1(0) &= f(0,0)+f(0,1) = .1+.2 = .3 \\ f_1(1) &= f(1,0)+f(1,1) = .4+.2 = .6 \\ f_1(2) &= f(2,0)+f(2,1) = .1+0 = .1 \end{aligned}$$

(d) Since

$$f_2(0) = f(0,0)+f(1,0)+f(2,0) = .1+.4+.1 = .6,$$

the conditional probability distribution of X_1 given $X_2 = 0$ is

$$\begin{aligned} f_1(0|0) &= \frac{f(0,0)}{f_2(0)} = \frac{.1}{.6} = \frac{1}{6} \\ f_1(1|0) &= \frac{f(1,0)}{f_2(0)} = \frac{.4}{.6} = \frac{4}{6} \\ f_1(2|0) &= \frac{f(2,0)}{f_2(0)} = \frac{.1}{.6} = \frac{1}{6} \end{aligned}$$

5.72 (a) The independent random variables X_1 and X_2 have the same probability distribution $b(x;2,.3)$. Hence the joint probability distribution of X_1 and X_2 is

$$\begin{aligned} f(x_1,x_2) &= b(x_1;2,.3)\cdot b(x_2;2,.3) = \binom{2}{x_1} .3^{x_1}.7^{2-x_1} \cdot \binom{2}{x_2} .3^{x_2}.7^{2-x_2} \\ &= \binom{2}{x_1}\binom{2}{x_2} .3^{x_1+x_2}.7^{4-x_1-x_2} \end{aligned}$$

where $x_1 = 0,1,2$, and $x_2 = 0,1,2$.

(b)

$$P(X_1 < X_2) = f(0,1)+f(0,2)+f(1,2)$$

$$= \binom{2}{0}\binom{2}{1}.3^1.7^3 + \binom{2}{0}\binom{2}{2}.3^2.7^2$$
$$+ \binom{2}{1}\binom{2}{2}.3^3.7^1$$
$$= .2058 + .0441 + .0378 = .2877$$

5.73 (a) $P(X_1 < 1, X_2 < 1) = F(1,1)$

$$= \int_0^1 \int_0^1 x_1 x_2 dx_2 dx_1 = \frac{1}{2}\int_0^1 x_1 dx_1 = \frac{x_1^2}{4} \Big|_0^1 = 1/4$$

(b) The probability that the sum is less than 1 is given by:

$$\int_0^1 \int_0^{1-x_1} x_1 x_2 dx_2 dx_1 = (1/2)\int_0^1 x_1(1-x_1)^2 dx_1$$
$$= (1/2)(x_1^4/4 - 2x_1^3/3 + x_1^2/2)\Big|_0^1 = (1/2)(1/4 - 2/3 + 1/2) = 1/24$$

5.74

$$f_1(x_1) = \int_{-\infty}^{\infty} f(x_1, x_2)dx_2 = \int_0^2 x_1 x_2 dx_2 = 2x_1 \quad \text{for } 0 < x_1 < 1$$

$$f_2(x_2) = \int_{-\infty}^{\infty} f(x_1, x_2)dx_1 = \int_0^1 x_1 x_2 dx_1 = x_2/2 \quad \text{for } 0 < x_2 < 2$$

5.75 The joint distribution function is given by:

$$F(x_1, x_2) = \int_0^{x_1} \int_0^{x_2} s_1 s_2 ds_2 ds_1 = \frac{1}{2}\int_0^{x_1} x_2^2 s_1 ds_1 = x_1^2 x_2^2/4$$

for $0 < x_1 < 1$ and $0 < x_2 < 2$. Thus, the distribution function is

$$F(x_1,x_2)=\begin{cases}0 & x_1\le 0 \text{ or } x_2\le 0\\ x_1^2x_2^2/4 & 0<x_1<1 \text{ and } 0<x_2<2\\ x_2^2/4 & x_1\ge 1 \text{ and } 0<x_2<2\\ x_1^2 & 0<x_1<1 \text{ and } x_2\ge 2\\ 1 & x_1\ge 1 \text{ and } x_2\ge 2\end{cases}$$

The distribution function of X_1 is

$$F_1(x_1)=\int_0^{x_1} f_1(s_1)ds_1=\int_0^{x_1} 2s_1ds_1=x_1^2 \quad \text{for } 0<x_1<1.$$

Thus,

$$F_1(x_1)=\begin{cases}0 & x_1\le 0\\ x_1^2 & 0<x_1<1\\ 1 & x_1\ge 1\end{cases}$$

Similarly,

$$F_2(x_2)=\int_0^{x_2} f_2(s_2)ds_2=\frac{1}{2}\int_0^{x_2} s_2ds_2=x_2^2/4 \quad \text{for } 0<x_2<2.$$

Thus,

$$F_2(x_2)=\begin{cases}0 & x_2\le 0\\ x_2^2/4 & 0<x_2<2\\ 1 & x_2\ge 2\end{cases}$$

It is easy to see that $F_1(x_1)\cdot F_2(x_2)=F(x_1,x_2)$. Thus, the random variables are independent.

5.76 $P(.2<X<.5,\ .4<Y<.6)$

$$\begin{aligned}&= \int_{.2}^{.5}\int_{.4}^{.6}\frac{6}{5}(x+y^2)dydx \;=\; \int_{.2}^{.5}\left[\frac{6}{5}(yx+\frac{y^3}{3})\,|_{.4}^{.6}\right]dx\\ &= \frac{6}{5}\int_{.2}^{.5}(.2x+.152/3)dx \;=\; \frac{6}{5}\left(.2x^2/2\,|_{.2}^{.5}+.152x/3\,|_{.2}^{.5}\right)\end{aligned}$$

$$= .04345$$

5.77 The joint distribution function is given by

$$F(x,y) = \int_0^x \int_0^y \frac{6}{5}(u+v^2)dvdu = \frac{3x^2y}{5} + \frac{2xy^3}{5} \quad \text{for } 0<x<1,\ 0<y<1$$

Thus, the joint distribution is

$$F(x,y) = \begin{cases} 0 & x \le 0 \text{ or } y \le 0 \\ (3/5)x^2y + (2/5)xy^3 & 0<x<1,\ 0<y<1 \\ (3/5)y + (2/5)y^3 & x \ge 1,\ 0<y<1 \\ (3/5)x^2 + (2/5)x & 0<x<1,\ y \ge 1 \\ 1 & x \ge 1,\ y \ge 1 \end{cases}$$

The probability of the region in the preceeding exercise is given by

$$\begin{aligned} F(.5,.6) - F(.2,.6) - F(.5,.4) + F(.2,.4) &= .1332 - .03168 - .0728 + .01472 \\ &= .04344 \end{aligned}$$

5.78 The marginal density for X is given by

$$f_1(x) = \int_0^1 \frac{6}{5}(x+y^2)dy = \frac{6}{5}\left(xy + \frac{y^3}{3}\right)\Big|_0^1 = \frac{6x}{5} + \frac{2}{5}$$

The marginal density for Y is given by

$$f_2(y) = \int_0^1 \frac{6}{5}(x+y^2)dx = \frac{6}{5}\left(\frac{x^2}{2} + y^2x\right)\Big|_0^1 = \frac{3}{5} + \frac{6y^2}{5}$$

(a) Thus,

$$P(X > .8) = \int_{.8}^1 (6x/5 + 2/5)dx = (3x^2/5 + 2x/5)\ \Big|_{.8}^1 = .296$$

(b)

$$P(Y < .5) = \int_0^{.5} (3/5 + 6y^2/5)dy = (3/5)(.5) + (2/5)(.5)^3 = .35$$

5.79 (a) By definition

$$f_1(x|y) = \frac{f(x,y)}{f_2(y)} = \begin{cases} (x+y^2)/(\frac{1}{2}+y^2) & \text{for } 0 < y < 1,\ 0 < x < 1 \\ 0 & \text{elsewhere.} \end{cases}$$

(b) Thus,

$$f_1(x|.5) = \frac{x + .5^2}{\frac{1}{2} + .5^2} = \begin{cases} 4x/3 + 1/3 & \text{for } 0 < x < 1 \\ 0 & \text{elsewhere.} \end{cases}$$

(c) The mean is given by

$$\int_0^1 x(4x/3 + 1/3)dx = (4x^3/9 + x^2/6)\ |_0^1 = 11/18$$

5.80 (a) The joint density is:

$$f(x_1, x_2) = \begin{cases} (2/3)(x_1 + 2x_2) & 0 < x_1 < 1,\ 0 < x_2 < 1 \\ 0 & \text{elsewhere} \end{cases}$$

In the example, it was shown that the conditional density of the first random variable given that the second takes on the value x_2 is

$$f_1(x_1|x_2) = \frac{2x_1 + 4x_2}{1 + 4x_2}$$

When $x_2 = .25$, $f_1(x_1|x_2 = .25) = x_1 + 1/2$.

(b) The marginal density of X_1 is

$$f_1(x_1) = \int_0^1 f(x_1, x_2)dx_2 = \int_0^1 \frac{2}{3}(x_1 + 2x_2)dx_2$$

$$= \frac{2}{3}(x_1x_2 + x_2^2)\Big|_{x_2=0}^{x_2=1} = \frac{2}{3}(x_1 + 1)$$

Thus,

$$f_2(x_2|x_1) = \frac{2(x_1 + 2x_2)/3}{2(x_1+1)/3} = \frac{x_1 + 2x_2}{x_1 + 1} \quad \text{for } 0 < x_1 < 1,\ 0 < x_2 < 1.$$

5.81 (a) To find k, we must integrate the density and set it equal to 1. Thus,

$$\int_0^1 \int_0^2 \int_0^\infty k(x+y)e^{-z} dz dy dx \int_0^1 \int_0^2 k(x+y) dy dx$$

$$= \int_0^1 k(2x+2)dx = 3k = 1$$

Thus, $k = 1/3$.

(b) $P(X < Y,\ Z > 1)$

$$= \frac{1}{3}\int_0^1 \int_x^2 \int_1^\infty (x+y)e^{-z} dz dy dx = \frac{1}{3e}\int_0^1 \int_x^2 (x+y) dy dx$$
$$= \frac{1}{3e}\int_0^1 (2x + 2 - 3x^2/2)dx = 5/(6e) = .3066$$

5.82 (a)

$$f_1(x) = \int_0^2 \int_0^\infty \frac{1}{3}(x+y)e^{-z} dz dy = 2(x+1)/3$$
$$f_2(y) = \int_0^1 \int_0^\infty \frac{1}{3}(x+y)e^{-z} dz dx = (y+1/2)/3$$
$$f_3(z) = \int_0^1 \int_0^2 \frac{1}{3}(x+y)e^{-z} dy dx = e^{-z}$$

Thus,

$$f_1(x)f_2(y)f_3(z) = \frac{1}{3}(y+1/2)\cdot\frac{2}{3}(x+1)\cdot e^{-z}$$
$$= \frac{2}{9}(xy + y + x/2 + 1/2)e^{-z}$$

$$\neq \frac{1}{3}(x+y)e^{-z} = f(x,y,z)$$

Thus, the three random variables are not independent.

(b)

$$\begin{aligned} f(x,y) &= \int_0^\infty \frac{1}{3}(x+y)e^{-z}dz &= \frac{1}{3}(x+y) \\ f(x,z) &= \int_0^2 \frac{1}{3}(x+y)e^{-z}dy &= \frac{2}{3}(x+1)e^{-z} \\ f(y,z) &= \int_0^1 \frac{1}{3}(x+y)e^{-z}dx &= \frac{1}{3}(y+1/2)e^{-z} \end{aligned}$$

Since

$$f(x,y) = \frac{1}{3}(x+y) \neq f_1(x)f_2(y) = \frac{2}{9}(xy+y+x/2+1/2),$$

X and Y are not independent. Since

$$f(x,z) = \frac{2}{3}(x+1)e^{-z} = f_1(x)f_3(z),$$

X and Z are independent. Since

$$f(y,z) = \frac{1}{3}(y+1/2)e^{-z} = f_2(y)f_3(z),$$

Y and Z are independent.

5.83 (a) Notice that $f(x_1, x_2)$ can be factored into

$$\frac{1}{\sqrt{2\pi}\sigma} \exp\left[\frac{-1}{2\sigma^2}(x_1-\mu_1)^2\right] \cdot \frac{1}{\sqrt{2\pi}\sigma} \exp\left[\frac{-1}{2\sigma^2}(x_2-\mu_2)^2\right]$$

Thus, X_1 and X_2 are independent normal random variables. Thus,
$P(-8 < X_1 < 14, -9 < X_2 < 3) = P(-8 < X_1 < 14)P(-9 < X_2 < 3)$
$= (F((14-2)/10) - F((-8-2)/10))\,(F((3+2)/10) - F((-9+2)/10))$

$= (F(1.2) - F(-1))(F(.5) - F(-.7))$

$= (.8849 - .1587)(.6915 - .2420)$

$= (.7262)(.4495) = .3264$

(b) When $\mu_1 = \mu_2 = 0$ and $\sigma = 3$, the density is

$$f(x_1, x_2) = \frac{1}{2\pi\sigma} \exp\left[\frac{-1}{2\sigma^2}(x_1^2 + x_2^2)\right]$$

Let R be the region between the two circles. We need to find

$$\int_R \frac{1}{2\pi\sigma^2} \exp\left[\frac{-1}{2\sigma^2}(x_1^2 + x_2^2)\right] dx_1 dx_2$$

Changing to polar coordinates gives

$$\begin{aligned} &\int_{r=3}^{r=6} \int_0^{2\pi} \frac{1}{2\pi\sigma^2} \exp(-r^2/(2\sigma^2)) r d\theta dr \\ &= \int_3^6 \frac{1}{\sigma^2} \exp(-r^2/(2\sigma^2)) r dr \;=\; -\exp(-r^2/(2\sigma^2)) \,|_3^6 \\ &= e^{-1/2} - e^{-2} \;=\; .4712 \end{aligned}$$

5.84 Let R be the region for acceptable holes. Then

$$P(\text{hole will acceptable}) = \int_R \frac{1}{8\pi} \exp(-(x_1^2 + x_2^2)/8) dx_1 dx_2$$

Changing to polar coordinates gives

$$\int_{r=0}^{r=8} \int_0^{2\pi} \frac{1}{8\pi} e^{-r^2/8} r d\theta dr = \int_0^8 \frac{1}{4} e^{-r^2/8} r dr = -e^{-r^2/8} \,|_0^8 = 1 - e^{-8}$$

5.85 The expected value of $g(X_1, X_2)$ is

$$\begin{aligned} \int_{-\infty}^{\infty} g(x_1, x_2) f(x_1, x_2) dx_1 dx_2 &= \int_0^1 \int_0^2 (x_1 + x_2) x_1 x_2 dx_2 dx_1 \\ &= \int_0^1 (2x_1^2 + 8x_1/3) dx_1 \;=\; 2 \end{aligned}$$

5.86 The expected value of $g(X,Y)$ is given by

$$\int_0^1\int_0^1 \frac{6}{5}x^2y(x+y^2)dydx = \frac{6}{5}\int_0^1(\frac{x^3}{2}+\frac{x^2}{4})dx = \frac{6}{5}(\frac{1}{8}+\frac{1}{12}) = \frac{1}{4}$$

5.87 The area of the rectangle is $X \cdot Y$. Thus the mean of the area distribution is given by

$$\int_{L-a/2}^{L+a/2}\int_{W-b/2}^{W+b/2} \frac{xy}{ab}dydx = \int_{L-a/2}^{L+a/2} \frac{x}{a}Wdx = LW$$

The variance is given by

$$\begin{aligned}\int_{L-a/2}^{L+a/2}\int_{W-b/2}^{W+b/2} \frac{x^2y^2}{ab}dydx - (WL)^2 &= (W^2+b^2/12)(L^2+a^2/12) - (WL)^2 \\ &= \frac{1}{12}((aW)^2+(bL)^2+(ab)^2/12)\end{aligned}$$

5.88

$$f_1(x_1|x_2) = \frac{f(x_1,x_2)}{f_2(x_2)} \quad , \qquad f_2(x_2|x_1) = \frac{f(x_1,x_2)}{f_1(x_1)}$$

Thus,

$$f_1(x_1|x_2) = f_2(x_2|x_1)\frac{f_1(x_1)}{f_2(x_2)}$$

5.89 (a) $E(X_1+X_2) = E(X_1)+E(X_2) = 1+(-1) = 0.$

(b) $Var(X_1+X_2) = Var(X_1)+Var(X_2) = 5+5 = 10.$

5.90 (a) $E(X_1-X_2) = E(X_1)+(-1)E(X_2) = (-2)+(-1)5 = -7.$

(b) $Var(X_1-X_2) = Var(X_1)+(-1)^2Var(X_2) = 2+(-1)^2(5) = 7.$

5.91 (a) $E(X_1+2X_2-3) = E(X_1+2X_2)-3 = E(X_1)+2E(X_2)-3 = 1+2(-2)-3$ $= -6.$

(b) $Var(X_1+2X_2-3) = Var(X_1+2X_2) = Var(X_1)+2^2Var(X_2) = 5+2^2(5)$ $= 25.$

5.92 (a) When checking a single chip, the time saving is $Y = X_1 - X_2$. Hence the expected time saving is

$$E(Y) = E(X_1 - X_2) = E(X_1) - E(X_2) = 65 - 54 = 11 \text{ milliseconds}$$

(b) $E(200Y) = 200E(Y) = 200(11) = 2200$ milliseconds $= 2.2$ seconds

(c) $Var(Y) = Var(X_1 - X_2) = Var(X_1) + (-1)^2 Var(X_2) = 16 + (-1)^2 9 = 25$
$Var(200Y) = 200^2 Var(Y) = 200^2 5^2 = (1000)^2$.
Hence the standard deviations of Y and $200Y$ are 5 and 1000 milliseconds respectively.

5.93 (a) $E(X_1 + X_2 + \cdots + X_{20}) = E(X_1) + E(X_2) + \cdots + E(X_{20})$
$= 20(10) = 200$.

(b) $Var(X_1 + X_2 + \cdots + X_{20}) = Var(X_1) + Var(X_2) + \cdots + Var(X_{20})$
$= 20(3) = 60$.

5.94 (a) For any seven observations the normal-scores are those z_i, i=1,...,7 that satisfy $F(z_i) = i/(7+1)$. Thus,

$$\begin{aligned} &F(z_1) = .125 = F(-1.15), && F(z_2) = .25 = F(-.67), \\ &F(z_3) = .375 = F(-.32), && F(z_4) = .500 = F(0), \\ &F(z_5) = .625 = F(.32), && F(z_6) = .750 = F(.67), \\ &F(z_7) = .875 = F(1.15) && \end{aligned}$$

So, -1.15, -0.67, -0.32, 0, 0.32, 0.67, 1.15 are the normal-scores of any seven observations.

(b) The normal-scores plot using the seven observations 16, 10, 18, 27, 29, 19 and 17 is given in Figure 5.1.

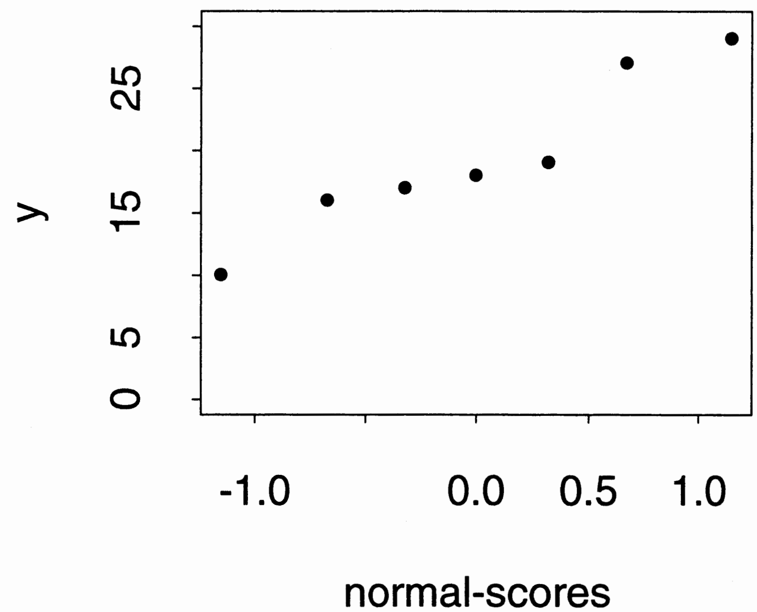


Figure 5.1: Normal-scores plot for Exercise 5.94b

5.95 (a) For any eleven observations the normal scores z_i, i=1, ..., 11, satisfy $F(z_i) = i/12$, thus using Table 3 the normal-scores are:

$$-1.38, -0.97, -0.67, -0.43, -0.21, 0, 0.21, 0.43, 0.67, 0.97, 1.38$$

(b) The observations on the times (sec.) between neutrinos are: .107, .196, .021, .283, .179, .854, .58, .19, 7.3, 1.18, 2.0. The normal-scores plot is given in Figure 5.2.

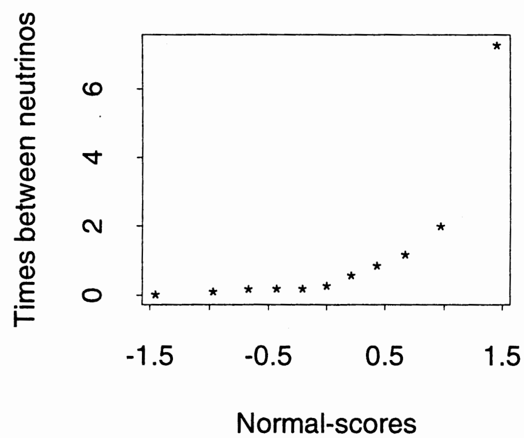


Figure 5.2: Normal-scores plot for Exercise 5.95b

5.96 (a) The normal-scores plot of the aluminum alloy strength data is given in Figure 5.3.

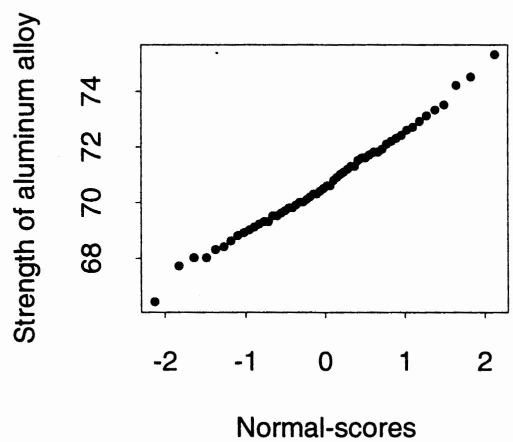


Figure 5.3: Normal-scores plot for Exercise 5.96a

(b) The normal-scores plot of the decay data is given in Figure 5.4.

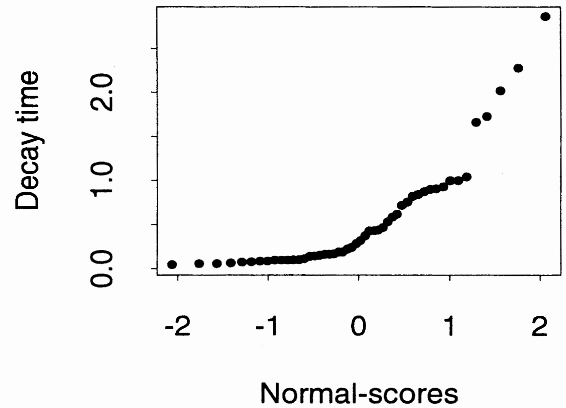


Figure 5.4: Normal-scores plots for Exercise 5.96b

5.97 (a) The normal-scores plots of the logarithmic, square root and fourth root transformations for the decay time data are given in Figures 5.5, 5.6 and 5.7, respectively.

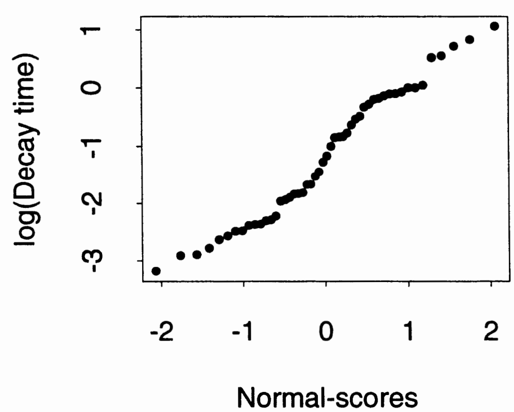


Figure 5.5: Normal-scores plot of the log(decay time) data. Exercise 5.97a

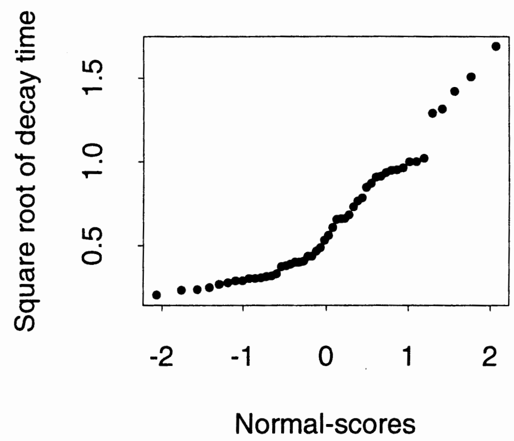


Figure 5.6: Normal-scores plot of square root of the decay time data. Exercise 5.97a

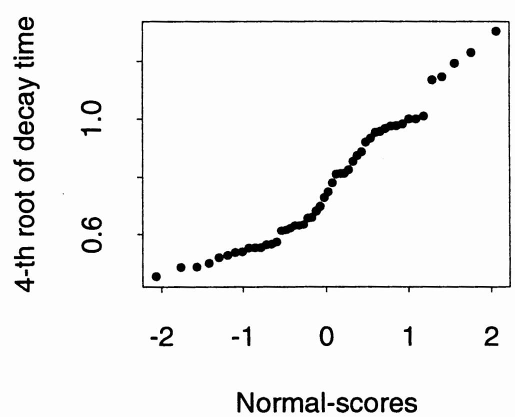


Figure 5.7: Normal-scores plot of fourth root of the decay time data. Exercise 5.97a

(b) The normal-scores plots of the logarithmic, square root and fourth root transformations for the interarrival time data are given in Figures 5.8, 5.9 and 5.10, respectively.

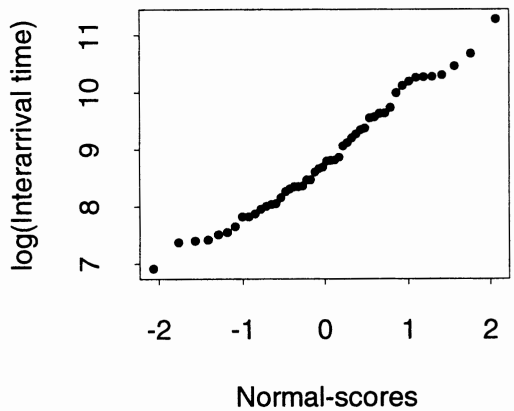


Figure 5.8: Normal-scores plot of the log(interarrival time) data. Exercise 5.97b

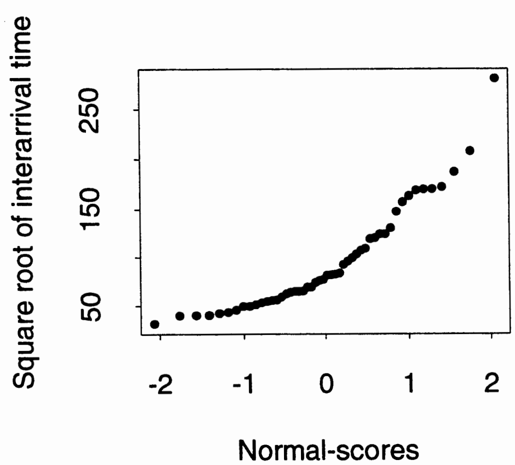


Figure 5.9: Normal-scores plot of square root of the interarrival time data. Exercise 5.97b

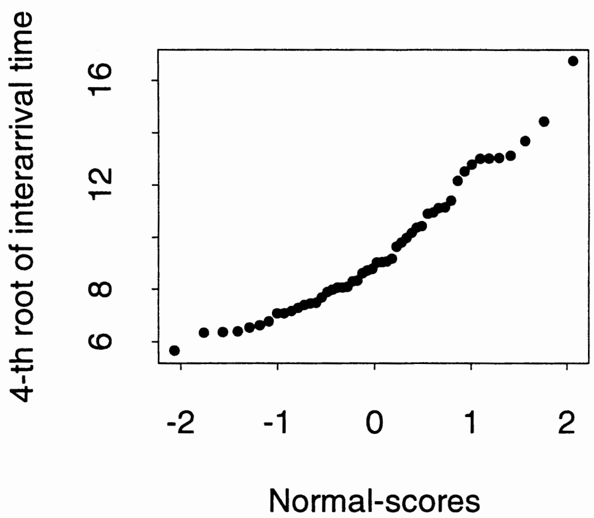


Figure 5.10: Normal-scores plot of fourth root of the interarrival time data. Exercise 5.97b

5.98 The simulated values of the Weibull random variable with $\alpha = .05$ and $\beta = 2.0$ are:

$x = (-20\ln(1-.26))^{\frac{1}{2}} = 2.45400,$ $\qquad x = (-20\ln(1-.77))^{\frac{1}{2}} = 5.42158,$

$x = (-20\ln(1-.12))^{\frac{1}{2}} = 1.59896.$

5.99 The time it takes a person to learn how to operate a certain machine is normal random variable with mean $\mu = 5.8$ and standard deviation $\sigma = 1.2$ and it takes two persons to operate the machine. Using Minitab , we generate two columns of normal variates having mean 5.8 and standard deviation 1.2. Each row represents a pair of workers.

ROW	C1	C2
1	6.16327	5.71118
2	5.37122	5.15066
3	5.21900	6.44433
4	4.73400	6.35715

The simulated times it takes four pairs to learn how to operate the machine are 6.16327, 5.37122, 6.44433 and 6.35715.

5.100 We generate the five exponential variates, with mean $\beta = 2$ in C1. Each row represents the durability of paint on one house.

ROW	C1
1	3.10639
2	0.96682
3	3.24836
4	2.83178
5	2.27078

(a) The time of the first failure is .96682

(b) The time of the fifth failure is 3.24836

5.101 (a) The density is

$$f(x) = \begin{cases} .3e^{-.3x} & x > 0 \\ 0 & \text{elsewhere} \end{cases}$$

Thus, the corresponding distribution function is

$$F(x) = \begin{cases} \int_0^x .3e^{-.3s} ds = -e^{-.3s} \mid_0^x = 1 - e^{-.3x} & \text{for } x > 0 \\ 0 & \text{elsewhere} \end{cases}$$

(b) We solve $u = F(x)$ for x. Since $u = F(x) = 1 - e^{-.3x}$, so $e^{-.3x} = 1 - u$ or $-.3x = \ln(1-u)$. The solution is then $x = -\ln(1-u)/.3$

5.102 (a) The density is

$$f(x) = \begin{cases} \alpha\beta x^{\beta-1} e^{-\alpha x^\beta} & \text{for } x > 0 \\ 0 & \text{elsewhere} \end{cases}$$

Thus, the corresponding distribution function is

$$F(x) = \begin{cases} \int_0^x \alpha\beta s^{\beta-1} e^{-\alpha s^\beta} ds = -e^{-\alpha s^\beta} \mid_0^x = 1 - e^{-\alpha x^\beta} & \text{for } x > 0 \\ 0 & \text{elsewhere} \end{cases}$$

(b) We solve $u = F(x)$ for x. Since $u = F(x) = 1 - e^{-\alpha x^\beta}$, so $e^{-\alpha x^\beta} = 1 - u$ or $-\alpha x^\beta = \ln(1-u)$. The solution is then $x = (-(1/\alpha)\ln(1-u))^{1/\beta}$.

5.103 Let Z_1, Z_2 be independent standard normal random variables. Under a change of polar coordinates, $Z_1 = R\cos(\Theta)$, $Z_2 = R\sin(\Theta)$, the joint density of R and Θ is

$$f(r,\theta) = \begin{cases} re^{-r^2/2}\frac{1}{2\pi} & 0 < \theta < 2\pi, r > 0 \\ 0 & \text{elsewhere} \end{cases}$$

(a) The marginal distribution of Θ is

$$f_2(\theta) = \int_0^\infty f(r,\theta)dr = \frac{1}{2\pi}\int_0^\infty re^{-r^2/2}dr$$

$$= \frac{1}{2\pi}(-e^{-r^2/2}) \mid_0^\infty = \frac{1}{2\pi}.$$

Hence Θ has uniform distribution on $(0,\ 2\pi)$. The marginal distribution of R is

$$f_1(r) = \frac{1}{2\pi}\int_0^{2\pi} re^{-r^2/2}d\theta = \frac{1}{2\pi}re^{-r^2/2}\theta \mid_0^{2\pi} = re^{-r^2/2} \quad \text{for } r > 0$$

Thus, R has Weibull distribution with $\alpha = 1/2$ and $\beta = 2$. Since

$$f_1(r)f_2(\theta) = f(r,\theta)$$

R and Θ are independent.

(b) Let $U_1 = \Theta/2\pi$ and $U_2 = 1 - e^{-R^2/2}$, then U_1 and U_2 are independent since

R and Θ are independent. The distribution function of U_1 is

$$F_1(u_1) = P(U_1 < u_1) = P(\Theta < 2\pi u_1) = \frac{2\pi u_1}{2\pi} = u_1 \quad \text{for } 0 < u_1 < 1$$

Thus, U_1 has uniform distribution on (0, 1). The distribution function of U_2 is

$$\begin{aligned} F_2(u_2) &= P(U_2 < u_2) = P(1 - e^{-R^2/2} < u_2) \\ &= P(R^2 < -2\ln(1-u_2)) = P(R < (-2\ln(1-u_2))^{1/2}) \\ &= -e^{-r^2/2}\Big|_0^{(-2\ln(1-u_2))^{1/2}} = u_2 \end{aligned}$$

Thus, U_2 has a uniform distribution on (0, 1).

(c) Since $1 - U_2 = e^{-R^2/2}$ and $U_1 = \Theta/2\pi$, we have

$$Z_1 = R\cos\Theta = \sqrt{-2\ln e^{-R^2/2}}\cos\Theta = \sqrt{-2\ln(1-U_2)}\cos(2\pi U_1)$$

$$Z_2 = R\sin\Theta = \sqrt{-2\ln e^{-R^2/2}}\sin\Theta = \sqrt{-2\ln(1-U_2)}\sin(2\pi U_1)$$

Note that $1 - U_2$ also has a uniform distribution on (0, 1). Hence $\ln U_2$ can be used in place of $\ln(1-U_2)$ in above equations, when we use independent uniform random variables U_1 and U_2 to generate independent standard normal variables Z_1 and Z_2. This completes the proof.

5.104 The eight exponential variates we generated, with $\beta = .2$, are:

```
0.053320   0.025485   0.071669   0.377028   0.208014   0.026851
   0.433405   0.747732
```

5.105 (a) Histogram of the time of the 100 first failures is

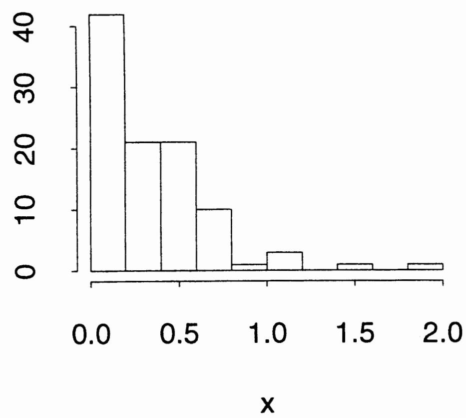


Figure 5.11: Histogram of the 100 first failure times. Exercise 5.105a.

(b) Histogram of the time of the 100 fifth failures is given in Figure 5.12.

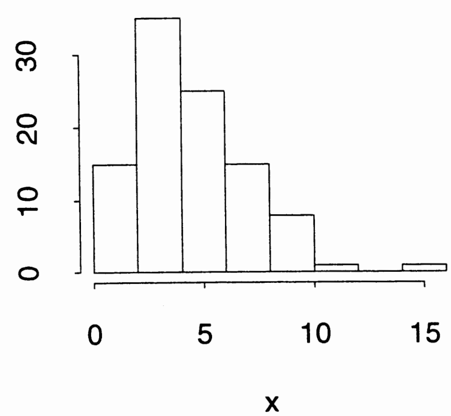
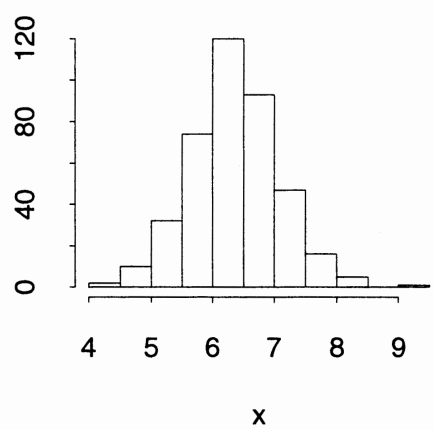


Figure 5.12: Histogram of the 100 fifth failure times. Exercise 5.105b.

5.106 Eight generated values for the normal random variable with $\mu = 123$ and $\sigma = 23.5$ are:

139.529, 159.167, 129.411, 143.942, 176.539, 88.797, 89.581, 196.666

5.107 (a) The histogram of the 400 learning times for the pairs of operators is



(b) The histogram of the 100 values representing the time to train four pairs of operators is given in Figure 5.13.

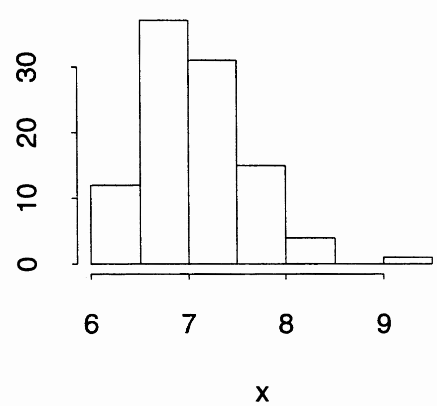


Figure 5.13: Histogram for Exercise 107b.

5.108 Let X be a random variable with density function

$$f(x) = \begin{cases} k(1-x^2) & \text{for } 0 < x < 1 \\ 0 & \text{elsewhere} \end{cases}$$

To find k we need to integrate $f(x)$ from 0 to 1 and set it equal to 1. Since,

$$\int_0^1 k(1-x^2)dx = k(x - x^3/3)\mid_0^1 = k(1-1/3) = k(2/3) = 1,$$

we have $k = 3/2$.

(a)

$$P(.1 < X < .2) = \frac{3}{2}\int_{.1}^{.2}(1-x^2)dx = \frac{3}{2}(x - x^3/3)\mid_{.1}^{.2}$$

$$= \frac{3}{2}(.2 - .008/3 - .1 + .001/3) = \frac{3}{2}\frac{.293}{3} = \frac{.293}{2} = .1465$$

(b) $P(X > .5) = (3/2)(x - x^3/3)\mid_{.5}^{1} = (3/2)(1 - 1/3 - .5 + (.5^3)/3) = 5/16$

5.109 The distribution function is given by:

$$F(x) = \int_{-\infty}^{x} f(s)ds = \int_0^x \frac{3}{2}(1-s^2)ds = \frac{3}{2}(x - x^3/3)$$

Thus,

(a) $P(X < .3) = F(.3) = (3/2)(.3 - .3^3/3) = .4365$

(b) $P(.4 < X < .6) = F(.6) - F(.4) = (3/2)(.6 - .6^3/3) - (3/2)(.4 - .4^3/3)$

$= .792 - .568 = .224.$

5.110 (a) P(error will be between -0.03 and 0.04)

$$= \int_{-.03}^{.04} 25dx = \int_{-.02}^{.02} 25dx = 1$$

(b) $P(\text{error will be between } -0.005 \text{ and } 0.005) = 25(.01) = .25$

5.111

$$\mu = \int_0^1 xf(x)dx = \frac{3}{2}\int_0^1 x(1-x^2)dx = \frac{3}{2}\left(\frac{x^2}{2} - \frac{x^4}{4}\right)\Big|_0^1 = \frac{3}{8}$$

Next,

$$\mu_2' = \frac{3}{2}\int_0^1 x^2(1-x^2)dx = \frac{3}{2}\left(\frac{x^3}{3} - \frac{x^5}{5}\right)\Big|_0^1 = .2$$

so the variance is equal to

$$\sigma^2 = \mu_2' - \mu^2 = .2 - \left(\frac{3}{8}\right)^2 = .0594$$

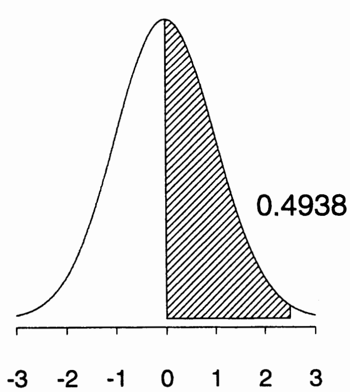
5.112 Let Z be a random variable with the standard normal distribution.

(a) $P(0 < Z < 2.5) = F(2.5) - .5 = .9938 - .5 = .4938$

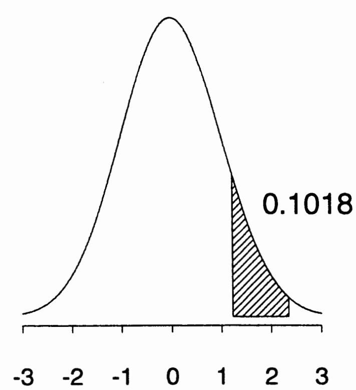
(b) $P(1.22 < Z < 2.35) = F(2.35) - F(1.22) = .9906 - .8888 = .1018$

(c) $P(-1.33 < Z < -.33) = F(-.33) - F(-1.33)$
$= .3707 - .0918 = .2789$

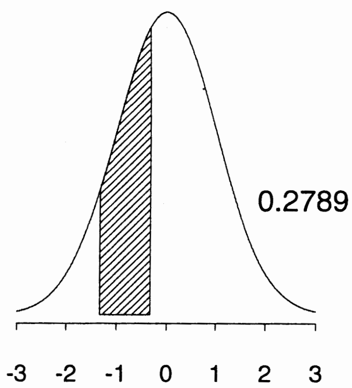
(d) $P(-1.60 < Z < 1.80) = F(1.80) + F(-1.60) = .9641 - .0548 = .9093$



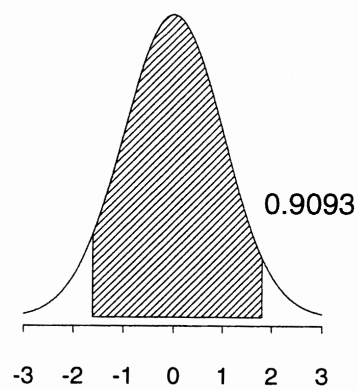
5.112 (a) z



5.112 (b) z



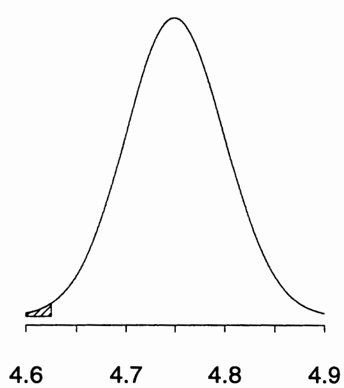
5.112 (c) z



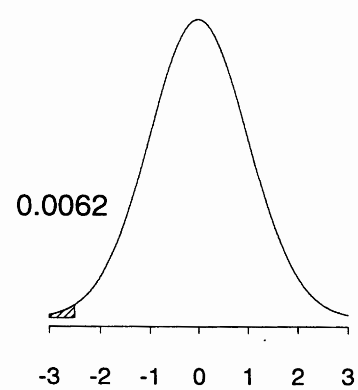
5.112 (d) z

5.113 Let X be a normal random variable with $\mu = 4.76$ and $\sigma = .04$

(a) $P(X < 4.66) = F((4.66 - 4.76)/.04) = F(-2.5) = .0062$



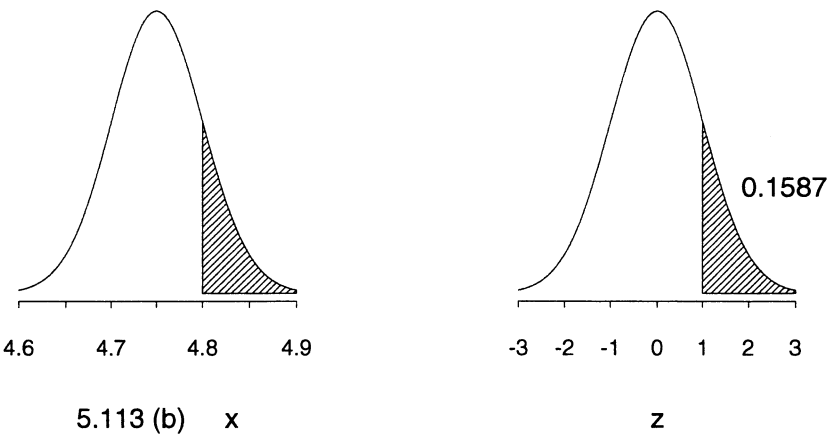
5.113 (a) x



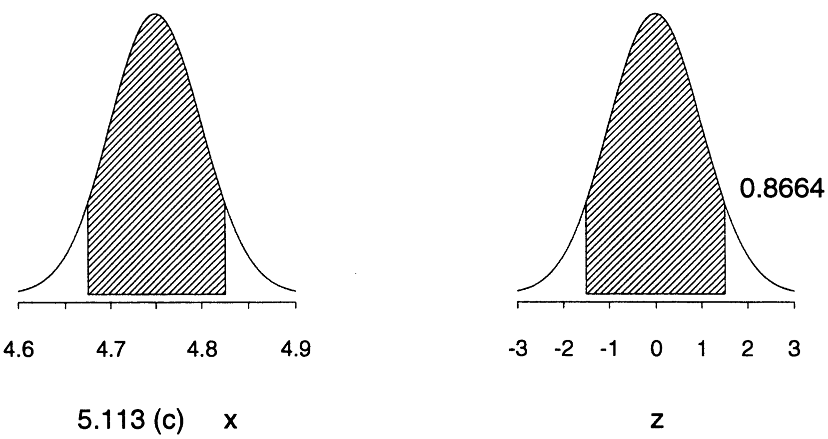
z

(b) $P(X > 4.8) = 1 - F((4.8 - 4.76)/.04) = 1 - F(1) = 1 - .8413 = .1587$

(c) $P(4.7 < X < 4.82) = F(4.82 - 4.76)/.04) - F((4.7 - 4.76)/.04)$
$= F(1.5) - F(-1.5) = .9332 - .0668 = .8664$



5.113 (b)



5.113 (c)

5.114 (a) $F(z_{.10}) = .90 = F(1.28)$, thus $z_{.10} = 1.28$

(b) $F(z_{.001}) = .999 = F(3.09)$, thus $z_{.001} = 3.09$

5.115 $Q_1 = -\sigma z_{.25} + \mu = -27(.675) + 102 = 83.775$

$Q_2 = \sigma z_{.5} + \mu = \mu = 102$

$Q_3 = \sigma z_{.25} + \mu = 27(.675) + 102 = 120.225$

5.116 The density function is

$$f(x) = \begin{cases} \frac{1}{\sqrt{2\pi}\beta} x^{-1} \exp\left[-(\ln x - \alpha)^2/2\beta^2\right] & x > 0, \beta > 0 \\ 0 & \text{elsewhere} \end{cases}$$

(a)

$$\begin{aligned} P(X > 200) &= 1 - P(X < 200) = 1 - \int_0^{200} \frac{1}{\sqrt{2\pi}\beta} x^{-1} e^{\frac{-1}{2\beta^2}(\ln x - \alpha)^2} dx \\ &= 1 - \int_{-\infty}^{\ln(200)} \frac{1}{\sqrt{2\pi}\beta} e^{-(y-\alpha)^2} dy = 1 - F(\frac{\ln(200) - \alpha}{\beta}) \\ &= 1 - F(\frac{\ln(200) - 8.85}{1.03}) = 1 - F(-3.448) \\ &= F(3.448) = .9997 \end{aligned}$$

(b)

$$P(X < 300) = F(\frac{\ln(300) - 8.85}{1.03}) = F(-3.05) = .0011$$

5.117 The density function is

$$f(x) = \begin{cases} .25e^{-.25x} & x > 0 \\ 0 & \text{elsewhere} \end{cases}$$

(a) P(time to observe a particle is more than 200 microseconds)

$= -e^{-.25x} |_{.2}^{\infty} = e^{-.05} = .951$

(b) P(time to observe a particle is less than 10 microseconds)

$= 1 - e^{-.0025} = 1 - .9975 = .0025$

5.118 The normal-scores plot of the suspended solids data is given in Figure 5.14.

5.119 The normal-scores plot of the velocity of light data is given in Figure 5.15.

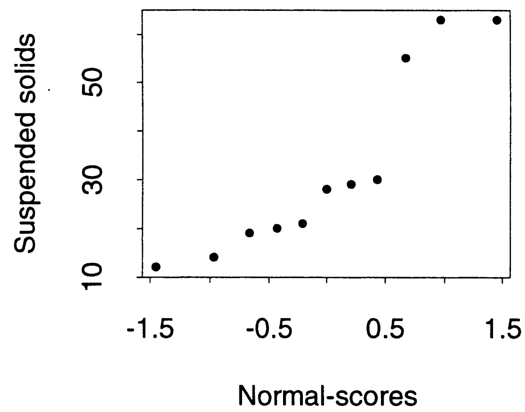


Figure 5.14: Normal-scores plot of the suspended solids data. Exercise 5.118.

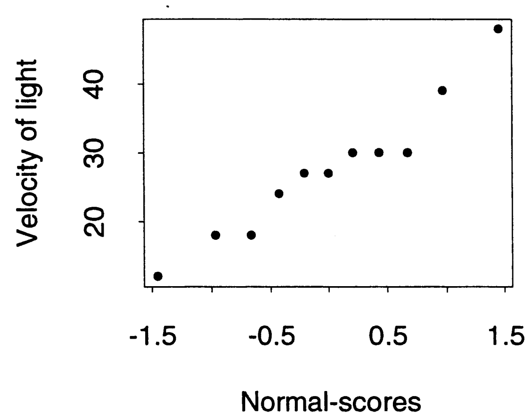


Figure 5.15: Normal-scores plot of the velocity of the light data. Exercise 5.119.

5.120 Let X be the gross sales volume, having the gamma distribution with $\alpha = 100\sqrt{n}$ and $\beta = \frac{1}{2}$ If the sales costs are 5,000 dollars per salesman then the expected profit is $E(P) = EX - 5n = \alpha\beta - 5n = 50\sqrt{n} - 5n$ which is maximized at $n = 25$.

5.121 Let X be the strength of a support beam, having the Weibull distribution with $\alpha = .02$ and $\beta = 3.0$.

$$P(X > 4.5) = \int_{4.5}^{\infty} (.02)3x^2 e^{-.02x^3}\,dx$$

Using the change of variable $u = x^3$, we have

$$P(X > 4.5) = \int_{(4.5)^3}^{\infty} .02e^{-.02u}\,du = e^{-.02(4.5)^3} = .1616$$

5.122 Let (X, Y) have the joint density function

$$f(x,y) = \begin{cases} .04e^{-.2x-.2y} & \text{for } x > 0, y > 0 \\ 0 & \text{elsewhere} \end{cases}$$

(a)

$$f_1(x) = \int_0^{\infty} .04e^{-.2x-.2y}\,dy = .2e^{-.2x} \quad \text{for} \quad x > 0$$

$$f_2(y) = \int_0^{\infty} .04e^{-.2x-.2y}\,dx = .2e^{-.2y} \quad \text{for} \quad y > 0$$

(b)

$$\begin{aligned} &\int_0^{\infty}\int_0^{\infty} (x+y)(.04)e^{-.2x-.2y}\,dx\,dy \\ &= \int_0^{\infty}\int_0^{\infty} x(.04)e^{-.2x-.2y}\,dy\,dx + \int_0^{\infty}\int_0^{\infty} y(.04)e^{-.2x-.2y}\,dx\,dy \\ &= \int_0^{\infty} x(.2)e^{-.2x}\left(\int_0^{\infty} .2e^{-.2y}\,dy\right)dx \\ &\quad + \int_0^{\infty} y(.2)e^{-.2y}\left(\int_0^{\infty} .2e^{-.2x}\,dx\right)dy \end{aligned}$$

$$= \int_0^\infty x(.2)e^{-.2x}dx + \int_0^\infty y(.2)e^{-.2y}dy = \frac{1}{.2} + \frac{1}{.2} = 10.$$

(c) Here $E(X) = 1/.2 = E(Y)$, so $E(X+Y) = E(X) + E(Y) = 10$.

5.123 (a) The independent random variables X_1 and X_2 have the same probability distribution $b(x; 2, .7)$. Hence the joint probability distribution of X_1 and X_2 is

$$\begin{aligned} f(x_1, x_2) &= b(x_1; 2, .7) \cdot b(x_2; 2, .7) \\ &= \binom{2}{x_1} .7^{x_1} .3^{2-x_1} \cdot \binom{2}{x_2} .7^{x_2} .3^{2-x_2} \\ &= \binom{2}{x_1}\binom{2}{x_2} .7^{x_1+x_2} .3^{4-x_1-x_2} \end{aligned}$$

where $x_1 = 0, 1, 2$, and $x_2 = 0, 1, 2$.

(b)

$$\begin{aligned} P(X_1 < X_2) &= f(0,1) + f(0,2) + f(1,2) \\ &= \binom{2}{0}\binom{2}{1} .7^1 .3^3 + \binom{2}{0}\binom{2}{2} .7^2 .3^2 \\ &\quad + \binom{2}{1}\binom{2}{2} .7^3 .3^1 \\ &= .0378 + .0441 + .2058 = .2877 \end{aligned}$$

5.124 (a) $E(3X_1 + 5X_2 + 2) = E(3X_1 + 5X_2) + 2 = 3E(X_1) + 5E(X_2) + 2$
$= 3(-5) + 5(1) + 2 = -8$.

(b) $Var(3X_1 + 5X_2 + 2) = Var(3X_1 + 5X_2) = 3^2 Var(X_1) + 5^2 Var(X_2)$

$$= 3^2(3) + 5^2(4) = 127.$$

5.125 (a) $E(X_1 + X_2 + \cdots + X_{30}) = E(X_1) + E(X_2) + \cdots + E(X_{30})$

$= 30(-5) = -150.$

(b) $Var(X_1 + X_2 + \cdots + X_{30}) = Var(X_1) + Var(X_2) + \cdots + Var(X_{30})$

$= 30(2) = 60.$