Chapter 4

PROBABILITY DISTRIBUTIONS

4.1 Let N be the number of suitable lasers.

$$
\begin{aligned}
P(N=0) &= 1/2 & \qquad P(N=3) &= 3/12 = 1/4 \\
P(N=1) &= 2/12 = 1/6 & \qquad P(N=4) &= 2/12 = 1/6 \\
P(N=2) &= 3/12 = 1/4 & \qquad P(N=5) &= 1/12.
\end{aligned}
$$

Thus, the distribution can be tabulated as :

N	0	1	2	3	4	5
Prob	1/12	1/6	1/4	1/4	1/6	1/12

4.2 This problem satisfies the binomial assumptions with $n = 4$ and $p = 1/2$. Thus the distribution is

$$P(\text{Total number of heads}) = \binom{4}{x}(\frac{1}{2})^x(\frac{1}{2})^{4-x} = \binom{4}{x}(\frac{1}{2})^4$$

for $x = 0, 1, 2, 3, 4$. Therefore

$$P(0) = 1/16, \quad P(1) = 1/4, \quad P(2) = 6/16, \quad P(3) = 1/4, \quad P(4) = 1/16.$$

4.3 (a) No. $\sum_{i=1}^{4} f(i) \ = \ 1.04 > 1$.

(b) Yes. $0 \le f(i) \le 1$, and $\sum_{i=1}^{4} f(i) \ = \ 1$.

(c) No. $f(3) < 0$.

4.4 (a) Yes. $0 \le f(i) \le 1$, and $\sum_{i=0}^{5} f(i) \ = \ 1$.

(b) No. $f(3) \ = \ -4/6 < 0$.

(c) Yes. $0 \le f(i) \le 1$, and $\sum_{i=3}^{6} f(i) \ = \ 1$.

(d) No. $\sum_{i=1}^{5} f(i) \ = \ 20/25 < 1$.

4.5 Using the identity

$$(x-1)\sum_{i=0}^{n} x^i \ = \ x^{n+1} - 1$$

or

$$\sum_{i=0}^{n} x^i \ = \ \frac{x^{n+1}-1}{x-1},$$

we have

$$\sum_{x=0}^{4} \frac{k}{2^x} \ = \ k\frac{(\frac{1}{2})^{4+1}-1}{\frac{1}{2}-1} \ = \ \frac{31k}{16}.$$

This must equal 1, so $k = 16/31$.

4.6

$$\begin{aligned} F(x) &= \sum_{i=0}^{x} f(i) \ = \ \sum_{i=0}^{x} \frac{k}{2^i} \ = \ k\,\frac{(\frac{1}{2})^{x+1}-1}{\frac{1}{2}-1} \\ &= \frac{16}{31}\,\frac{(\,1-(\frac{1}{2})^{x+1}\,)}{\frac{1}{2}} \ = \ \frac{32}{31}\,(1-(\frac{1}{2})^{x+1}) \end{aligned}$$

4.7

$$\begin{aligned}
b(x;n,p) &= \binom{n}{x} p^x(1-p)^{n-x} = \frac{n!}{x!\,(n-x)!}\; p^x(1-p)^{n-x} \\
&= \frac{n!}{(n-x)!\,x!}\;(1-p)^{n-x}\,p^x = \binom{n}{n-x}(1-p)^{n-x}\,p^x \\
&= b(n-x;n,1-p)
\end{aligned}$$

4.8 Using the result in Exercise 4.7,

$$\begin{aligned}
B(x;n,p) &= \sum_{i=0}^{x} b(i;n,p) = \sum_{i=0}^{x} b(n-i;n,1-p) \\
&= \sum_{u=n-x}^{n} b(u;n,1-p) = 1-\sum_{u=0}^{n-x-1} b(u;n,1-p) \\
&= 1-B(n-x-1;n,p)
\end{aligned}$$

4.9 (a) Assumptions appear to hold. Success is a home with a TV tuned to mayor's speech. The probability of success is the proportion of homes around city having a TV tuned to the mayor's speech.

(b) The binomial assumptions do not hold because the probability of a serious violation for the second choice depends on which plant is selected first.

4.10 (a) Success is DVD player fails. Trials are independent but the number of trials is not fixed. Therefore, the binomial distribution does not apply.

(b) Assumptions may not hold because some cavities occur on two adjacent teeth on their surfaces that touch each other. Success is when tooth has a cavity but the results for two adjacent teeth may be dependent. Binomial assumptions do not hold.

4.11 (a) Success is person has a cold. Colds are typically passed around in families so trials would not be independent. Therefore, the binomial distribution does not apply.

(b) Success means projector does not work properly. The binomial assumptions do not hold because the probability of a success for the second choice depends on which projector is selected first.

4.12 From Table 1:

(a) $B(8; 16, .4) = .8577$.

(b) $b(8; 16, .4) = (.8577) - (.7161) = .1416$.

(c) $B(9; 12, .6) = .9166$.

(d) $b(9; 12, .6) = (.9166) - (.7747) = .1419$.

(e) $\sum_{k=6}^{20} b(k; 20, .15) = 1 - B(5; 20, .15) = 1 - (.9327) = .0673$.

(f) $\sum_{k=6}^{9} b(k; 9, .7) = 1 - B(5; 9, .7) = 1 - (.2703) = .7297$.

4.13 From Table 1:

(a) $B(7; 19, .45) = .3169$.

(b) $b(7; 19, .45) = (.3169) - (.1727) = .1442$.

(c) $B(8; 10, .95) = .0861$.

(d) $b(8; 10, .95) = (.0861) - (.0115) = .0746$.

(e) $\sum_{k=4}^{10} b(k; 10, .35) = 1 - B(3; 10, .35) = 1 - (.5138) = .4862$.

$B(4; 9, .3) - B(1; 9, .3) = (.9012) - (.1960) = .7052$.

4.14 We need to find

$$\begin{aligned} P(3 \text{ or more need repair}) &= \sum_{x=3}^{20} b(x; 20, .1) = 1 - B(2; 20, .1) \\ &= 1 - (.6769) = .3231 \end{aligned}$$

Since this value is quite large, we probably cannot reject the manufacturer's claim.

4.15

$$b(2; 4, .75) = \binom{4}{2} (.75)^2 (.25)^{4-2} = .2109.$$

4.16

$$b(4; 12, .4) = \binom{12}{4} (.4)^4 (.6)^8 = 495(.4)^4(.6)^8 = .2128.$$

4.17 (a) $1 - B(11;\ 15,\ .7) = 1 - .7031 = .2969.$

(b) $B(6;\ 15,\ .7) = .0152.$

(c) $b(10;\ 15,\ .7) = B(10;\ 15,\ .7) - B(9;\ 15,\ .7) = (.4845) - (.2784) = .2061.$

4.18 (a) $b(1;\ 12,\ .05) = B(1;\ 12,\ .05) - B(0;\ 12,\ .05) = (.8816) - (.5404) = .3412.$

(b) $B(2;\ 12,\ .05) = .9804.$

(c) $1 - B(1;\ 12,\ .05) = 1 - (.8816) = .1184.$

4.19 (a) $P(18 \text{ are ripe}) = (.9)^{18} = .1501.$

(b) $1 - B(15;\ 18,\ .9) = 1 - .2662 = .7338.$

(c) $B(14;\ 18,\ .9) = .0982.$

4.20 (a) The probability that 1 or more components in a sample of 15 is defective when the true probability of being good is .95 is

$$1 - b(0; 15, .05) = 1 - (.95)^{15} = .5367.$$

(b) The probability that 0 are defective when the true probability is .90 is $b(0; 15, .10)$ $= .2059$.

(c) When the true probability is .90, we have $b(0; 15, .20) = .0352.$

4.21 (a) $B(2;\ 16,\ .05\ ) = .9571$

(b) $B(2;\ 16,\ .10\ ) = .7892$

(c) $B(2;\ 16,\ .15\ ) = .5614$

(d) $B(2;\ 16,\ .20\ ) = .3518$

4.22 The probabilities are:

i	$b(i; 10, .7)$
0	.0000
1	.0001
2	.0014
3	.0090
4	.0368
5	.1029
6	.2001
7	.2668
8	.2335
9	.1211
10	.0282

The probability histogram is given in Figure 4.1.

4.23 For this problem, we need to use the hypergeometric distribution. The probability is given by:

$$h(2; 6, 8, 18) = \frac{\binom{8}{2}\binom{10}{4}}{\binom{18}{6}} = \frac{(8!)\ (10!)\ (12!)}{2\ (6!)\ (4!)\ (18!)} = .3167$$

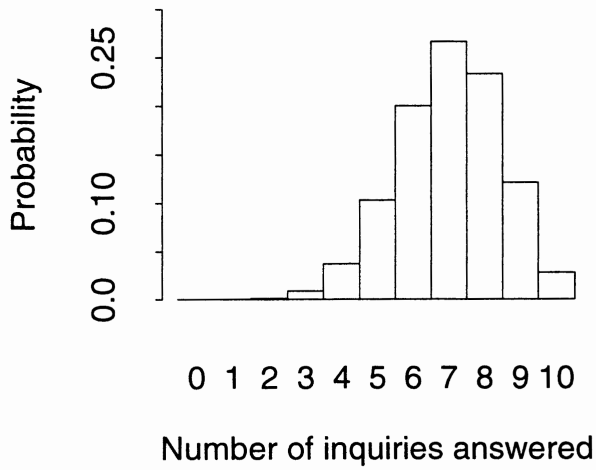

Figure 4.1: Probability Histogram for Exercise 4.22.

4.24 We need to use the hypergeometric distribution. The probability is given by:

$$h(3;4,9,12) = \frac{\binom{9}{3}\binom{3}{1}}{\binom{12}{4}} = \frac{4 \cdot 7}{11 \cdot 5} = .5091$$

4.25 Using the hypergeometric distribution,

(a)

$$h(0;3,6,24) = \frac{\binom{6}{0}\binom{18}{3}}{\binom{24}{3}} = \frac{816}{2024} = .4032$$

(b)

$$h(1;3,6,24) = \frac{\binom{6}{1}\binom{18}{2}}{\binom{24}{3}} = \frac{6 \cdot 153}{2024} = .4536$$

(c) The probability that at least 2 have defects is 1 minus the sum of the probabilities of none, and 1 having defects.

$$1 - (.4032 + .4536) = .1432$$

4.26 Using the hypergeometric distribution for x = number of buildings that violate the code,

(a)

$$h(0;3,6,18) = \frac{\binom{6}{0}\binom{12}{4}}{\binom{18}{4}} = \frac{1 \cdot 495}{3060} = .1618$$

(b)

$$h(1;4,6,18) = \frac{\binom{6}{1}\binom{12}{3}}{\binom{18}{4}} = \frac{6 \cdot 220}{3060} = .4314$$

(c)

$$h(2;4,6,18) = \frac{\binom{6}{2}\binom{12}{2}}{\binom{18}{4}} = \frac{15 \cdot 66}{3060} = .3235$$

(d) This is 1 minus the sum of probabilities that 0, 1, or 2 of the buildings violate the code

$$1 - (.1618 + .4314 + .3235) = .0833.$$

4.27 (a)

$$P(\text{none in west}) = h(0;3,7,16) = \frac{\binom{7}{0}\binom{9}{3}}{\binom{16}{3}} = \frac{1 \cdot 84}{560} = .15$$

(b)

$$P(\text{all in west}) = h(3;3,7,16) = \frac{\binom{7}{3}\binom{9}{0}}{\binom{16}{3}} = \frac{35 \cdot 1}{560} = .0625$$

4.28 (a)

$$P(\text{one bad alarm}) \quad = \quad h(1;3,5,120)$$

$$\doteq \frac{\binom{5}{1}\binom{115}{2}}{\binom{120}{3}} = \frac{5 \cdot 6555}{280,840} = .1167$$

(b)

$$P(\text{one bad alarm}) = \binom{3}{1}(\frac{5}{120})^1(\frac{115}{120})^2 = .1148$$

4.29 (a)

$$P(5 \text{ are union members}) = \frac{\binom{240}{5}\binom{60}{3}}{\binom{300}{8}} = .1470$$

(b)

$$P(5 \text{ are union members}) = \binom{8}{5}(\frac{240}{300})^4(\frac{60}{300})^3 = .1468$$

4.30 The binomial probabilities are

```
    PDF;
    BINOMIAL WITH n=27 p = .47 ;

 BINOMIAL WITH N =  27  P = 0.470000
    K          P( X = K)
    2            0.0000
    3            0.0001
    4            0.0004
```

```
     5          0.0016
     6          0.0052
     7          0.0138
     8          0.0305
     9          0.0571
    10          0.0912
    11          0.1249
    12          0.1477
    13          0.1512
    14          0.1340
    15          0.1030
    16          0.0685
    17          0.0393
    18          0.0194
    19          0.0081
    20          0.0029
    21          0.0009
    22          0.0002
    23          0.0000
```

4.31 The cumulative binomial probabilities are

```
  CDF;
  BINOMIAL n = 27 p = .47 .

BINOMIAL WITH N =  27  P = 0.470000
   K  P( X LESS OR = K)
   2          0.0000
   3          0.0001
   4          0.0005
   5          0.0021
   6          0.0072
   7          0.0210
   8          0.0515
   9          0.1086
  10          0.1998
  11          0.3247
  12          0.4724
  13          0.6236
  14          0.7576
```

15	0.8607
16	0.9292
17	0.9685
18	0.9879
19	0.9960
20	0.9989
21	0.9997
22	1.0000

4.32 (a) The mean μ is given by

$$\mu = 0(.4) + 1(.3) + 2(.2) + 3(.1) = 1$$

(b) The variance σ^2 is given by

$$\sigma^2 = (0-1)^2(.4) + (1-1)^2(.3) + (2-1)^2(.2) + (3-1)^2(.1) = 1$$

4.33 Using the computing formula:

$$\begin{aligned} \sigma^2 &= \mu_2' - \mu^2 \quad \text{with} \quad \mu = 1 \\ \mu_2' &= 0^2(.4) + 1^2(.3) + 2^2(.2) + 3^2(.1) = 2 \end{aligned}$$

Thus,

$$\sigma^2 = 2 - 1^2 = 1.$$

4.34 (a) The mean μ is given by

$$\mu = 0(.17) + 1(.29) + 2(.27) + 3(.16) + 4(.07) + 5(.03) + 6(.01) = 1.8$$

(b) The variance σ^2 is given by

$$\sigma^2 = (0-1.8)^2(.17) + (1-1.8)^2(.29) + (2-1.8)^2(.27) + (3-1.8)^2(.16)$$

$$+(4-1.8)^2(.07)+(5-1.8)^2(.03)+(6-1.8)^2(.01)=1.8$$

Thus the standard deviation is $\sigma = \sqrt{1.8} = 1.34$.

4.35 Using the computing formula:

$$\sigma^2 = \mu_2' - \mu^2, \qquad \mu = 1.8$$

$$\mu_2' = 0^2(.17)+1^2(.29)+2^2(.27)+3^2(.16)+4^2(.07)+5^2(.03)+6^2(.01) = 5.04$$

Thus,

$$\sigma^2 = 5.04 - (1.8)^2 = 1.8.$$

The standard deviation is $\sigma = \sqrt{1.8} = 1.34$.

4.36 Using the formula

$$\sum_{i=0}^{n} i = \frac{1}{2}n(n+1)$$

and

$$\sum_{i=0}^{n} i^2 = \frac{1}{6}n(n+1)(2n+1)$$

we find

$$\mu = \frac{1}{n}\sum_{i=0}^{n} i = \frac{n+1}{2}$$

and

$$\begin{aligned}\sigma^2 = \mu_2' - \mu^2 &= \frac{1}{n}\frac{1}{6}n(n+1)(2n+1) - [\,\frac{(n+1)}{2}\,]^2 \\ &= \frac{(n+1)(n-1)}{12} = \frac{(n^2-1)}{12}.\end{aligned}$$

4.37 (a) The mean for the binomial distribution with $n = 4$ and $p = 7$ can be calculated from the following table:

i	0	1	2	3	4
$b(i;4,.7)$	.0081	.0756	.2646	.4116	2401

Thus,

$$\begin{aligned} \mu &= 0(.0081)+1(.0756)+2(.2646)+3(.4116)+4(.2401)=2.8 \\ \mu_2' &= 0^2(.0081)+1^2(.0756)+2^2(.2646)+3^2(.4116)+4^2(.2401)=8.68 \\ \sigma^2 &= 8.68-(2.8)^2=.84. \end{aligned}$$

(b) $\mu = np = 4(.7) = 2.8.$

$\sigma^2 = np(1-p) = 4(.7\ )(.3) = .84.$

4.38 (a) The mean is given by:

$$\mu = 0\cdot\frac{1}{32}+1\cdot\frac{5}{32}+2\cdot\frac{10}{32}+3\cdot\frac{10}{32}+4\cdot\frac{5}{32}+5\cdot\frac{1}{32}=2.5$$

(b) $\mu = np = 5(.5) = 2.5.$

4.39 (a) The variance is given by:

$$\sigma^2 = (0-2.5)^2\frac{1}{32}+(1-2.5)^2\frac{5}{32}+(2-2.5)^2\frac{10}{32}+(3-2.5)^2\frac{10}{32}$$

$$+(4-2.5)^2\frac{5}{32}+(5-2.5)^2\frac{1}{32}=\frac{40}{32}=1.25$$

(b) To use the computing formula we need:

$$\mu_2' = 0^2\cdot\frac{1}{32}+1^2\cdot\frac{5}{32}+2^2\cdot\frac{10}{32}+3^2\cdot\frac{10}{32}+4^2\cdot\frac{5}{32}+5^2\cdot\frac{1}{32}=\frac{240}{32}=7.5$$

Thus

$$\sigma^2 = 7.5-(2.5)^2 = 1.25.$$

(c) The special formula for the binomial variance is:

$$\sigma^2 = np(1-p) = 5(.5)(.5) = 1.25$$

4.40 (a) First we need to find $b(i; 20, .95)$, $i = 0$ to 20. These are given in the following table:

i	0	1	2	3	4	5	6
$b(i; 20, .95)$	.0000	.0000	.0000	.0000	.0000	.0000	.0000

i	7	8	9	10	11	12	13
$b(i; 20, .95)$	.0000	.0000	.0000	.0000	.0000	.0000	.0000

i	14	15	16	17	18	19	20
$b(i; 20, .95)$	.0003	.0022	.0133	.0596	.1887	.3773	.3585

Thus,

$$\begin{aligned} \mu &= (0+1+\cdots+13)(.0000) + 14(.0003) + 15(.0022) + 16(.0133) \\ &\quad +17(.0596) + 18(.1887) + 19(.3773) + 20(.3585) = 19.0000 \end{aligned}$$

and

$$\begin{aligned} \mu_2' &= (0^2+1^2+\cdots+13^2)(.0000) + 14^2(.0003) + 15^2(.0022) + \\ &\quad 16^2(.0133) + 17^2(.0596) + 18^2(.1887) + 19^2(.3773) + 20^2(.3585) \\ &= 361.9496 \end{aligned}$$

Therefore,

$$\sigma^2 = 361.9496 - (19.0000)^2 = .9496.$$

(b) The special formulas for the binomial give:

$$\mu = np = 20(.95) = 19$$

$$\sigma^2 = np(1-p) = 20(.95)(.05) = .95$$

4.41 Since all of these random variables have binomial distributions,

(a)

$$\mu = np = 676(.5) = 338$$

$$\sigma^2 = np(1-p) = 676(.5)(.5) = 169$$

$$\sigma = 13$$

(b)

$$\mu = np = 720 \cdot \frac{1}{6} = 120$$

$$\sigma^2 = np(1-p) = 720 \cdot \frac{1}{6}\frac{5}{6} = 100$$

$$\sigma = 10$$

(c)

$$\mu = np = 600(.04) = 24$$

$$\sigma^2 = np(1-p) = 600(.04)(.96) = 23.04$$

$$\sigma = 4.8$$

(d)

$$\mu = np = 800(.65) = 520$$

$$\sigma^2 = np(1-p) = 800(.65)(.35) = 182$$

$$\sigma = 13.49$$

4.42 (a) The probabilities are:

$$P(X=0)=\frac{\binom{4}{0}\binom{4}{3}}{\binom{8}{3}}=\frac{4}{56}$$

$$P(X=1)=\frac{\binom{4}{1}\binom{4}{2}}{\binom{8}{3}}=\frac{24}{56}$$

$$P(X=2)=\frac{\binom{4}{2}\binom{4}{1}}{\binom{8}{3}}=\frac{24}{56}$$

$$P(X=3)=\frac{\binom{4}{3}\binom{4}{0}}{\binom{8}{3}}=\frac{4}{56}$$

Thus,

$$\begin{aligned}\mu &= 0\cdot\frac{4}{56}+1\cdot\frac{24}{56}+2\cdot\frac{24}{56}+3\cdot\frac{4}{56}=1.5\\ \sigma^2 &= (0-1.5)^2\cdot\frac{4}{56}+(1-1.5)^2\cdot\frac{24}{56}+(2-1.5)^2\cdot\frac{24}{56}\\ &\quad +(3-1.5)^2\cdot\frac{4}{56}=\frac{30}{56}=.5357\end{aligned}$$

(b) From the special formulas, we have

$$\mu = n \cdot \frac{a}{N} = 3 \cdot \frac{4}{8} = 1.5$$

$$\sigma^2 = \frac{n \cdot a(N-a)(N-n)}{N^2(N-1)} = \frac{3 \cdot 4 \cdot 4 \cdot 5}{8^2 \cdot 7} = \frac{240}{448} = .5357$$

4.43 The mean of the hypergeometric distribution is

$$\begin{aligned}\mu &= \sum_{x=0}^{n} x \frac{\binom{a}{x}\binom{N-a}{n-x}}{\binom{N}{n}} \\ &= \sum_{x=1}^{n} \frac{x\binom{a}{x}\binom{N-a}{n-x}}{\binom{N}{n}} \\ &= \frac{a}{\binom{N}{n}} \sum_{x=1}^{n} \binom{a-1}{x-1}\binom{N-a}{n-x}\end{aligned}$$

Let $u = x - 1$. Then,

$$\mu = \frac{a}{\binom{N}{n}} \sum_{u=0}^{n-1} \binom{a-1}{u}\binom{N-a}{n-1-u}$$

Using the identity given in the problem, we have

$$\mu = \frac{a\begin{pmatrix} N-1 \\ n-1 \end{pmatrix}}{\begin{pmatrix} N \\ n \end{pmatrix}} = \frac{an}{N}$$

The result holds for all n such that $0 \leq n \leq N$, because

$$\begin{pmatrix} a \\ x \end{pmatrix}\begin{pmatrix} N-a \\ n-x \end{pmatrix} = 0 \quad \text{if} \quad x > a \quad \text{or} \quad (n-x) > (N-a)$$

4.44 The tail probabilities and upper bounds are

No. of sd's	Upper bound of tail probabilities	Tail probabilities from binomial (16; 1/2)
1	1	.4544
2	1/4	.0768
3	1/9	.0042

The upper bound of the tail probability comes from

$$P(\mid X - \mu \mid \geq k\,\sigma) \leq \frac{1}{k^2}$$

where k is the number of standard deviations. An example of the calculation of the tail probabilities for the binomial distribution with $n = 16$ and $p = 1/2$ follows for the case $k = 2$,

$$\mu = np = 8, \qquad \sigma^2 = np(1-p) = 4, \qquad \sigma = 2.$$

Thus, we need $P(X \leq 4) + P(X \geq 12)$ when X has distribution $b(x; 16, 1/2)$. From Table 1, this is .0384 + 1 − .9616 = .0768 .

4.45 Since $\sigma = 0.002$ we have $0.006 = 3(0.0002)$, and $k = 3$.

$$P(\mid X - \mu \mid \leq 0.006)1 \;-\; P(\mid X - \mu \mid > 0.006) \geq 1 \;-\; \frac{1}{3^2} = \frac{8}{9}$$

4.46 In this case, $\mu = 67.5$, $\sigma^2 = 56.25$, and $\sigma = 7.5$. Thus, 30 and 105 are both 5 standard deviations from the mean. We need to find a bound for $P(|X - 67.5| \geq 5(7.5))$ which is $1/25 = .04$ by Chebyshev's theorem.

4.47

$$\mu = 1,000,000 \cdot \frac{1}{2} = 500,000.$$

$$\sigma^2 = 1,000,000 \cdot \frac{1}{2} \cdot \frac{1}{2} = 250,000 \quad , \quad \sigma = 500.$$

If the proportion is between .495 and .505, the number of heads must be between 495,000 and 505,000. These bounds are both within 10 standard deviations of the mean. We can apply Chebyshev's theorem with $k = 10$. Thus, the probability is greater than $1 - 1/100 = .99$.

4.48 We need to find k such that $1/k^2 = .01$. Thus, $k = 10$. Chebyshev's theorem states that,

$$P(\mid X - \mu \mid \geq k\sigma) \leq \frac{1}{k^2},$$

or

$$P(\mid \frac{X}{n} - \frac{\mu}{n} \mid \geq k\frac{\sigma}{n}) \leq \frac{1}{k^2},$$

Now $\mu = .5n$, $\sigma^2 = n/4$, $\sigma = \sqrt{n}/2$ Thus, we need to find n such that,

$$k\frac{\sigma}{n} = .04 \quad \text{or} \quad \frac{10\sqrt{n}/2}{n} = .04,$$

or

$$n = (\frac{5}{.04})^2 = 15,625.$$

4.49 (a) By definition of variance,

$$\begin{aligned}
\sigma^2 &= \sum_{\text{all } x} (x-\mu)^2 f(x) \\
&= \sum_{\text{all } x} (x^2 - 2x\mu + \mu^2) f(x) \\
&= \sum_{\text{all } x} x^2 f(x) - \sum_{\text{all } x} 2x\mu f(x) + \sum_{\text{all } x} \mu^2 f(x) \\
&= \mu_2' - 2\mu \sum_{\text{all } x} x f(x) + \mu^2 \sum_{\text{all } x} f(x) \\
&= \mu_2' - 2\mu \cdot \mu + \mu^2 \cdot 1 \\
&= \mu_2' - \mu^2
\end{aligned}$$

(b) Similarly,

$$\begin{aligned}
\mu_3 &= \sum_{\text{all } x} (x-\mu)^3 f(x) \\
&= \sum_{\text{all } x} (x^3 - 3x^2\mu + 3x\mu^2 - \mu^3) f(x) \\
&= \sum_{\text{all } x} x^3 f(x) - \sum_{\text{all } x} 3x^2\mu f(x) + \sum_{\text{all } x} 3x\mu^2 f(x) - \sum_{\text{all } x} \mu^3 f(x) \\
&= \mu_3' - 3\mu \sum_{\text{all } x} x^2 f(x) + 3\mu^2 \sum_{\text{all } x} x f(x) - \mu^3 \sum_{\text{all } x} f(x) \\
&= \mu_3' - 3\mu \cdot \mu_2' + 3\mu^2 \cdot \mu - \mu^3 \cdot 1 \\
&= \mu_3' - 3\mu\mu_2' + 2\mu^3
\end{aligned}$$

4.50

$$f(x+1;\lambda) = \frac{\lambda^{x+1}e^{-\lambda}}{(x+1)!} \quad \text{and} \quad f(x;\lambda) = \frac{\lambda^x e^{-\lambda}}{x!}$$

Thus,

$$\frac{f(x+1;\lambda)}{f(x;\lambda)} = \frac{\lambda^{x+1}e^{-\lambda}}{(x+1)!}\,\frac{x!}{\lambda^x e^{-\lambda}} = \frac{\lambda}{x+1}.$$

4.51 For $\lambda = 3$, $f(0;\lambda) = e^{-3} = .0498$. Thus, using

$$f(x+1;\lambda) = \frac{\lambda}{x+1} f(x;\lambda)$$

$$f(1;\lambda) = \frac{\lambda}{0+1} f(0;\lambda) = 3e^{-3} = .1494.$$

$$f(2;\lambda) = \frac{3}{2} \cdot 3e^{-3} = .2240.$$

and so forth. The values are given in the following table:

x	0	1	2	3	4
$f(x;3)$	.0498	.1494	.2240	.2240	.1680

x	5	6	7	8	9
$f(x;3)$	.1008	.0504	.0216	.0081	.0027

The probability histogram is given in Figure 4.2.

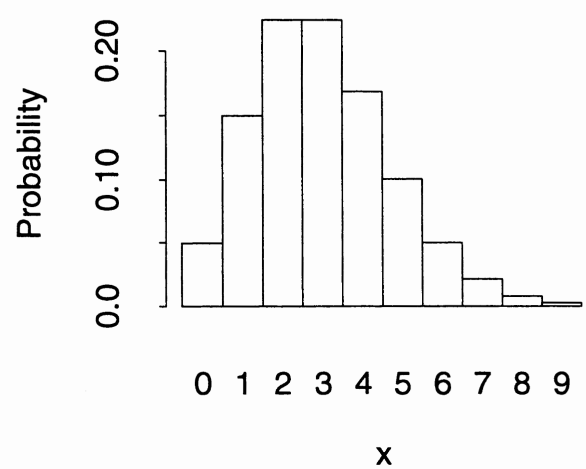

Figure 4.2: Probability Histogram for Exercise 4.51.

4.52 (a) $F(4;7) = .173$

(b) $f(4;7) = F(4;7) - F(3;7) = (.173) - (.082) = .091$

(c) $\sum_{k=6}^{19} f(k;8) = F(19;8) - F(5;8) = 1.0 - (.191) = .809$

4.53 (a) $F(9;12) = .242$

(b) $f(9;12) = \text{F}(\ 9;\ 12) - \text{F}(\ 8;\ 12) = (\ .242\ ) - (\ .155) = .087$

(c)

$$\sum_{k=3}^{12} f(k;7.5) = F(12;7.5) - F(2;7.5) = (.9573) - (.0203) = .937$$

where we have interpolated, by eye, between the entries for $\lambda = 7.4$ and $\lambda = 7.6$.

4.54 We use the approximation $b(x;n,p) \approx f(x;np)$. Thus

$$f(3;100(.03)) = f(3;3) = 3^3 e^{-3}/3! = .224.$$

4.55 (a) $n = 80$, $p = .06$, $np = 4.8$. Thus,

$$f(4;8.4) = F(4;4.8) - F(3;4.8) = (.476) - (.294) = .182$$

(b) $1 - F(2;4.8) = 1 - (.143) = .857$

(c)

$$\sum_{k=3}^{6} f(k;4.8) = F(6;4.8) - F(2;4.8) = (.791) - (.143) = .648$$

4.56 $n = 400$, $p = .008$, $np = 3.2$.

$$f(4;3.2) = F(4;3.2) - F(3;3.2) = (.781) - (.603) = .178$$

4.57 $1 - F(12;5.8) = 1 - .993 = .007$.

4.58 (a) $P($ at least one call $) = 1 - F(0; .6) = 1 - (\ .549) = .451.$

(b) λ for a 4-minute interval is 2.4 . Thus,

$$\begin{aligned} P(\text{at least 3 calls in a 4-minute interval}) &= 1 - F(2; 2.4) \\ &= 1 - (.570) = .430 \end{aligned}$$

4.59 (a) $P(\text{at most 4 in a minute}) = F(4;\ 1.5\ ) = .981.$

(b) $P(\text{at least 3 in 2 minutes}) = 1 - F(2; 3) = 1 - (.423) = .577.$

(c) $P(\text{at most 15 in 6 minutes}) = F(15; 9) = .978.$

4.60 Expected profit $= 600t - \sum_{x=0}^{\infty} 50x^2 f(x; .8t)$. Now,

$$\sum_{x=0}^{\infty} x^2 f(x; .8t) = \mu_2' = \sigma^2 + \mu^2$$

But, $\mu = .8t$ and $\sigma^2 = .8t$, so

$$\text{Expected profit} = 600t - 50(.8t + (.8t)^2) = 560t - 32t^2.$$

Taking the derivative, setting it equal to 0, and solving for t gives $t = 8.75$. Profits are maximized when $t = 8.75$ hours.

4.61 By definition,

$$b(1; x, p) = \begin{pmatrix} x \\ 1 \end{pmatrix} p^1(1-p)^{x-1} = xp^1(1-p)^{x-1}$$

so,

$$\frac{b(1; x, p)}{x} = p(1-p)^{x-1} = g(x; p).$$

(a) $g(12; .1) = b(1; 12, .1)/12 = (.6590 - .2824)/12 = .0314.$

(b) $g(10; .3) = b(1; 10, .3)/10 = (.1493 - .0282)/10 = .0121.$

4.62 This probability is given by

$$g(15; .05) = (.95)^{14}(.05) = .0244.$$

4.63 P (fails after 1,200 times)

$$= \sum_{x=1201}^{\infty} (1-p)^{x-1} p = \frac{(1-p)^{1200}\, p}{1-(1-p)} = (1-p)^{1200}$$

where $p = .001$. Thus,

$$P(\text{fails after 1,200 times }) = (.999)^{1200} = .3010.$$

4.64 This probability is given by

$$g(6; .2) = (.8)^5(.2) = .0655.$$

4.65 The Poisson process with $\alpha = 0.3$ applies.

(a) $\lambda = 0.3(10) = 3.$

$$f(2;3) = \frac{3^2}{2!} e^{-3} = 0.2240.$$

(b) We calculate $1 - f(0;3) - f(1;3) = 1 - 4e^{-3} = .8009$, or using Table 2,

$$1 - F(1;3) = 1 - 0.199 = 0.801.$$

(c) For either of the two 5-minute spans, the probability of exactly 1 customer is

$$f(1; 1.5) = \frac{(1.5)}{1!} e^{-1.5} = .3347$$

The two time intervals do not overlap so the counts are independent and we multiply the two probabilities

$$f(1;1.5) \times f(1;1.5) = (.3347) \times (.3347) = .1120$$

4.66 The Poisson process with $\alpha = 2$ applies.

(a) We use Table 2 with $\lambda = 2(2) = 4$.

$$f(5;4) = F(5:4) - F(4;4) = 0.785 - 0.629 = 0.156$$

(b)

$$1 - F(7;4) = 1 - 0.949 = 0.051.$$

(c) For the separate one hour periods, the probabilities are

$$f(2;2) = \frac{2^2}{2!}e^{-2} = .2707 \qquad f(3;2) = \frac{2^3}{3!}e^{-2} = .1804$$

The intervals do not overlap so the counts are independent and we multiply the two probabilities

$$f(2;2) \times f(3;2) = (.2707) \times (.1804) = .0488$$

4.67 The Poisson process with $\alpha = 0.6$ applies.

(a) $\lambda = 0.6(2) = 1.2$.

$$f(2;1.2) = \frac{(1.2)^2}{2!}e^{-1.2} = 0.1563.$$

(b) For the first hundred feet, the probability of exactly 1 flaw is

$$f(1;0.6) = \frac{.06}{1}e^{-0.6} = .0511$$

The intervals do not overlap so the counts are independent and we multiply the two probabilities

$$f(1;\lambda) \times f(1;\lambda) = (.3293) \times (.3293) = .1084$$

4.68 We know that

$$\sum_{x=1}^{\infty} p(1-p)^{x-1} = 1$$

Differentiating with respect to p gives

$$\sum_{x=1}^{\infty} (1-p)^{x-1} - \sum_{x=1}^{\infty} p(x-1)\,(1-p)^{x-2} = 0$$

Letting $u = x - 1$ gives

$$\sum_{u=0}^{\infty} (1-p)^{u} - \sum_{u=0}^{\infty} pu\,(1-p)^{u-1} = 0$$

The second term is μ , thus

$$\mu = \sum_{u=0}^{\infty} (1-p)^{u}$$

The right hand side is a geometric series with $0 \leq 1-p \leq 1$, so

$$\sum_{u=0}^{\infty} (1-p)^{u} = \frac{1}{1-(1-p)} = \frac{1}{p}$$

Thus, $\mu = 1/\text{p}$.

4.69

$$\begin{aligned}
\mu &= \sum_{x=0}^{\infty} xf(x,\lambda) = \sum_{x=0}^{\infty} x\,\frac{(\lambda)^x}{x!}e^{-\lambda} = \lambda \sum_{x=1}^{\infty} \frac{(\lambda)^{x-1}}{(x-1)!}e^{-\lambda} \\
&= \lambda \sum_{x=0}^{\infty} \frac{(\lambda)^x}{x!}e^{-\lambda} \;=\; \lambda \\
\mu_2' &= \sum_{x=0}^{\infty} x^2 f(x,\lambda) = \sum_{x=0}^{\infty} x^2\,\frac{(\lambda)^x}{x!}e^{-\lambda} = \sum_{x=1}^{\infty} \frac{x(\lambda)^x}{(x-1)!}e^{-\lambda}
\end{aligned}$$

$$= \lambda \sum_{x=0}^{\infty} (x+1) \frac{(\lambda)^x}{x!} e^{-\lambda} \quad = \quad \lambda(\lambda+1)$$

Thus,

$$\sigma^2 = \mu_2' - \mu^2 = \lambda(\lambda+1) - \lambda^2 = \lambda$$

4.70 The Poisson probabilities are

(a)

```
Poisson with mu = 2.73000

    x        P( X = x )
 2.00            0.2430
 3.00            0.2212
```

(b)

```
Poisson with mu = 4.33000

    x        P( X = x )
 2.00            0.1234
 3.00            0.1782
```

4.71 The cumulative Poisson probabilities are

(a)

```
Poisson with mu = 2.73000

    x       P( X <= x )
 2.00            0.4863
 3.00            0.7075
```

(b)

```
Poisson with mu = 4.33000

    x       P( X <= x )
 2.00            0.1936
 3.00            0.3718
```

4.72 To find this probability , we use the multinomial distribution with $n = 12$, $x_1 = 4$, $x_2 = 6$, $x_3 = 2$, $p_1 = .4$, $p_2 = .4$ and $p_3 = .2$, Thus the probability is given by

$$\frac{12!}{4!\ 6!\ 2!}(.4)^4\ (.4)^6\ (.2)^2 = .0581$$

4.73 To find this probability , we use the multinomial distribution with $n = 6$, $x_1 = 2$, $x_2 = 3$, $x_3 = 1$, $p_1 = 1/4$, $p_2 = 1/2$ and $p_3 = 1/4$, Thus the probability is given by

$$\frac{6!}{2!\ 3!\ 1!}(\frac{1}{4})^2\ (\frac{1}{2})^3\ (\frac{1}{4})^1 = .117$$

4.74 This probability is a multinomial with $n = 10$, $x_1 = 5$, $x_2 = 3$, $x_3 = 1$, $x_4 = 1$, $p_1 = .6$, $p_2 = .2$, $p_3 = .1$ and $p_4 = .1$, Thus the probability is given by

$$\frac{10!}{5!\ 3!\ 2!\ 1!}(.6)^5\ (.2)^3\ (.1)^1\ (.1)^1 = .0314$$

4.75 (a) Let $\sum_{i=1}^k a_i = N$ and $\sum_{i=1}^k x_i = n$. Then the formula for the probability of getting x_i objects of the i-th kind when there are a_i objects of the i-th kind is

$$\frac{\binom{a_1}{x_1}\binom{a_2}{x_2}\cdots\binom{a_k}{x_k}}{\binom{N}{n}} = \frac{\prod_{i=1}^k \binom{a_i}{x_i}}{\binom{N}{n}}$$

(b) Here $a_1 = 10$, $a_2 = 7$, $a_3 = 3$, with $n = 6$, $x_1 = 3$, $x_2 = 2$, and $x_3 = 1$. Thus the probability is

$$\frac{\binom{10}{3}\binom{7}{2}\binom{3}{1}}{\binom{20}{6}} = \frac{120 \cdot 21 \cdot 3}{38,760} = .1950$$

4.76 Let , for example, 0-4 represent heads, and 5-9 represent tails. Using the first column of Table 7, and starting in row 1, gives the following simulation:

Flip no.	1	2	3	4	5	6	7	8	9	10
Random digit	1	0	6	7	7	5	2	1	3	6
H or *T*	H	H	T	T	T	T	H	H	H	T
Flip no.	11	12	13	14	15	16	17	18	19	20
Random digit	9	4	8	8	1	2	3	5	1	1
H or *T*	T	H	T	T	H	H	H	T	H	H
Flip no.	21	22	23	24	25	26	27	28	29	30
Random digit	3	1	7	8	8	0	6	8	8	6
H or *T*	H	H	H	T	T	H	T	T	T	T
Flip no.	31	32	33	34	35	36	37	38	39	40
Random digit	4	8	3	6	2	1	1	9	0	0
H or *T*	H	T	H	T	H	H	H	T	H	H
Flip no.	41	42	43	44	45	46	47	48	49	50
Random digit	4	1	3	3	7	2	1	0	5	8
H or *T*	H	H	H	H	T	H	H	H	T	T
Flip no.	51	52	53	54	55	56	57	58	59	60
Random digit	2	5	7	6	6	8	9	5	9	5
H or *T*	H	T	T	T	T	T	T	T	T	T
Flip no.	61	62	63	64	65	66	77	68	69	70
Random digit	2	6	9	2	7	6	2	0	7	8
H or *T*	H	T	T	H	T	T	H	H	T	T
Flip no.	71	72	73	74	75	76	77	78	79	80
Random digit	8	5	2	7	3	2	2	1	2	7
H or *T*	T	T	H	T	H	H	H	H	H	T

Flip no.	81	82	83	84	85	86	87	88	89	90
Random digit	0	9	8	5	7	7	5	3	3	6
H or T	H	T	T	T	T	T	T	H	H	T
Flip no.	91	92	93	94	95	96	97	98	99	100
Random digit	0	7	7	3	5	3	0	6	1	8
H or T	H	T	T	H	T	H	H	T	H	T

4.77 We will use column 4 of Table 7 for this simulation. Starting in row 1, and omitting 0, 7, 8, and 9 gives:

Roll no.	1	2	3	4	5	6	7	8	9	10	11	12
Digit	6	2	5	5	1	6	3	5	6	2	6	2
Roll no.	13	14	15	16	17	18	19	20	21	22	23	24
Digit	2	4	3	3	4	5	5	3	4	2	6	1
Roll no.	25	26	27	28	28	30	31	32	33	34	35	36
Digit	3	4	2	3	4	6	5	4	1	5	2	5
Roll no.	37	38	39	40	41	42	43	44	45	46	47	48
Digit	2	4	4	6	1	5	4	1	3	5	6	4
Roll no.	49	50	51	52	53	54	55	56	57	58	59	60
Digit	6	6	6	1	2	5	1	5	1	6	4	3
Roll no.	61	62	63	64	65	66	67	68	69	70	71	72
Digit	4	1	1	5	2	5	4	5	4	4	5	5
Roll no.	73	74	75	76	77	78	79	80	81	82	83	84
Digit	4	5	2	5	4	6	3	4	2	1	1	5
Roll no.	85	86	87	88	89	90	91	92	93	94	95	96
Digit	1	5	1	6	5	6	3	2	2	3	6	1

Roll no.	97	98	99	100	101	102	103	104	105	106	107	108
Digit	2	6	2	1	2	2	6	5	5	2	1	6
Roll no.	109	110	111	112	113	114	115	116	117	118	119	120
Digit	3	1	4	2	6	3	3	1	3	6	6	1

Since the table ran out of random digits in column 4, we started over in column 8, row 1.

4.78 (a) The following table distributes the random numbers:

Number of sales	Probability	Cumulative probability	Random numbers
0	.14	.14	00–13
1	.28	.42	14–41
2	.27	.69	42–68
3	.18	.87	69–86
4	.09	.96	87–95
5	.04	1.00	96–99

(b) Using columns 1 and 2 of Table 7, and starting in row 1, gives:

Day	1	2	3	4	5	6	7	8	9	10	11	12	13
Random no.	13	04	65	79	76	51	29	14	36	65	99	48	82
No. of sales	0	0	2	3	3	2	1	1	1	2	5	2	3
Day	14	15	16	17	18	19	20	21	22	23	24	25	
Random no.	87	13	22	35	58	11	14	33	11	74	84	83	
No. of sales	4	0	1	1	2	0	1	1	0	3	3	3	

4.79 The random numbers are distributed in the following table:

Number of claims	Probability	Cumulative probability	Random numbers
0	.041	.041	000–040
1	.130	.171	041–170
2	.209	.380	171–379
3	.223	.603	380–602
4	.178	.781	603–780
5	.114	.895	781–894
6	.060	.955	895–954
7	.028	.983	955–982
8	.011	.994	983–993
9	.004	.998	994–997
10	.002	1.000	998-999

Using columns 5-7 of Table 7, and starting in row 1, gives:

Year	1	2	3	4	5	6	7	8	9	10
Random no.	118	243	202	654	693	785	055	418	564	679
No. of failed generators	1	2	2	4	4	5	1	3	3	4

Year	11	12	13	14	15	16	17	18	19	20
Random no.	608	095	706	919	912	295	705	920	632	449
No. of failed generators	4	1	4	6	6	2	4	6	4	3

4.80 The random numbers are distributed in the following table:

Number units made	Probability	Cumulative probability	Random numbers
3	.10	.10	00–09
4	.40	.50	10–49
5	.30	.80	50–79
6	.20	1.00	80–99

Number units sold	Probability	Cumulative probability	Random numbers
0	.05	.05	00–04
1	.10	.15	05–14
2	.30	.45	15–44
3	.30	.75	45–74
4	.10	.85	75–84
5	.05	.90	85–89
6	.05	.95	90–94
7	.04	.99	95–98
8	.01	1.00	99

Using columns 13-14 of Table 7 and starting from row 101, gives:

Day	1	2	3	4	5	6	7	8	9	10
Random no. 1	99	06	62	78	60	89	59	27	12	49
Random no. 2	75	38	04	01	92	54	75	96	76	24
No. made	6	3	5	5	5	6	5	4	4	4
No. sold	4	2	0	0	6	3	4	7	4	2
Profit	400	200	0	0	600	300	400	700	400	200
Inventory	2	3	8	13	12	15	16	13	13	15
Net profit	360	140	-160	-260	360	0	80	440	140	-100
Day	11	12	13	14	15	16	17	18	19	20
Random no. 1	28	97	45	74	80	20	13	94	45	50
Random no. 2	27	00	57	26	81	58	14	46	37	13
No. made	4	6	4	5	6	4	4	6	4	5
No. sold	2	0	3	2	4	3	1	3	2	1
Profit	100	0	300	200	400	300	100	300	400	100
Inventory	17	21	22	25	27	28	31	34	36	40
Net profit	-140	-420	-140	-300	-140	-260	-520	-380	-320	-700
Day	21	22	23	24	25	26	27	28	29	30
Random no. 1	68	10	88	07	59	60	15	30	75	98
Random no. 2	75	28	45	01	12	41	86	66	70	99
No. made	5	4	6	3	5	5	4	4	5	6
No. sold	4	2	3	0	1	2	5	3	3	8
Profit	400	200	300	0	100	200	500	300	300	800
Inventory	41	43	46	49	53	56	55	56	58	56
Net profit	-420	-660	-620	-980	-960	-920	-600	-820	-860	-320

Day	31	32	33	34	35	36	37
Random no. 1	15	05	47	53	80	62	06
Random no. 2	90	46	48	28	31	88	55
No. made	4	3	4	5	6	5	3
No. sold	6	3	3	2	2	5	3
Profit	600	300	300	200	200	500	300
Inventory	54	54	55	58	62	62	62
Net profit	-480	-780	-800	-960	-1040	-740	-940
Day	38	39	40	41	42	43	44
Random no. 1	68	57	48	16	31	79	17
Random no. 2	93	23	67	49	27	74	74
No. made	5	5	4	4	4	5	4
No. sold	6	2	3	3	2	3	3
Profit	600	200	300	300	200	300	300
Inventory	61	63	64	65	67	69	70
Net profit	-620	-1060	-980	-1000	-1140	-1080	-1100
Day	45	46	47	48	49	50	
Random no. 1	74	69	46	55	28	62	
Random no. 2	04	84	79	68	30	67	
No. made	5	5	4	5	4	5	
No. sold	0	4	4	3	2	3	
Profit	0	400	400	300	200	300	
Inventory	75	76	76	78	80	82	
Net profit	-1500	-1120	-1120	-1260	-1400	-1340	

4.81 (a) $P($ 2 or more defects $) = f(2) + f(3) = .03 + .01 = .04$.

(b) 0 is more likely since its probability $f(0) = .89$ is much larger than $f(1) = .07$.

4.82 (a) $P($ requests exceed number of rooms $) = f(3) + f(4) = .25 + .08 = .33$.

(b) $P($ requests less than number of rooms $) = f(0) + f(1) = .07 + .15 = .22$.

(c) If 1 room is added, to make a total of 3 rooms, $P($ requests exceed number of rooms $) = f(4) = .08$ which satisfies the requirement.

4.83 (a) $\mu = 0 \times .07 \; + \; 1 \times .15 \; + \; 2 \times .45 \; + \; 3 \times .25 \; + \; 4 \times .08 \; = \; 2.12$.

(a) We first calculate

$$0^2 \times .07 \; + \; 1^2 \times .15 \; + \; 2^2 \times .45 \; + \; 3^2 \times .25 \; + \; 4^2 \times .08 \; = \; 5.48$$

so variance $= 5.48 - (2.12)^2 = .9856$

(c) standard deviation $= \sqrt{.9856} = .9928$ rooms

4.84 (a) No. $\sum_{i=0}^{2} f(i) \; = \; 1.02 > 1$.

(b) Yes. $0 \le f(i) \le 1$, and $\sum_{i=0}^{2} f(i) \; = \; 1$.

(c) No. $f(2) \; < \; 0$.

4.85 (a) Yes. $0 \le f(i) \le 1$, and $\sum_{i=0}^{4} f(i) \; = \; 1$.

(b) Yes. $0 \le f(i) \le 1$, and $\sum_{i=-1}^{1} f(i) \; = \; 1$.

(c) No. $\sum_{i=0}^{3} f(i) \; = \; 1.5 > 1$

4.86 This probability is given by geometric distribution. The probability of a miss is $p = 1 - .90 = .10$.

$$g(7; .05) = (.9)^6 (.1) = .048.$$

4.87 (a)

$$b(3; 8, .2) = \binom{8}{3} (.2)^3 \, (.8)^5 = \frac{8!}{3! \, 5!} (.2)^3 \, (.8)^5 = .1468$$

(b) $B(3; 8, .2) - B(2; 8, .2) = .9437 - .7969 = .1468$

4.88 (a) $b(18; 20, .85) = B(18; 20, .85) - B(17; 20, .85) = .8244 - .5951 = .2293$

(b) $1 - B(14; 20, .85) = 1 - .0673 = .9327$

(c) $1 - B(1; 20, .15) = 1 - .1756 = .8244$

4.89 Using the hypergeometric distribution,

(a)

$$h(0; 2, 4, 15) = \frac{\binom{4}{0}\binom{11}{2}}{\binom{15}{2}} = \frac{1 \cdot 55}{105} = .5238$$

(b)

$$h(1; 2, 4, 15) = \frac{\binom{4}{1}\binom{11}{1}}{\binom{15}{2}} = \frac{4 \cdot 11}{105} = .4190$$

(c)

$$h(2; 2, 4, 15) = \frac{\binom{4}{2}\binom{11}{0}}{\binom{15}{2}} = \frac{6 \cdot 1}{105} = .0571$$

4.90 (a) The mean is given by:

$$\mu = 0(.216) + 1(.432) + 2(.288) + 3(.064) = 1.2.$$

(b) Using the special formula for the binomial mean

$$\mu = np = 3(.4) = 1.2$$

4.91 (a) The variance is given by:

$$\begin{aligned} \sigma^2 &= (0-1.2)^2(.216) + (1-1.2)^2(.432) + (2-1.2)^2(.288) + (3-1.2)^2(.064) \\ &= .72 \end{aligned}$$

(b) Using the special formula for the binomial variance

$$\sigma^2 = np(1-p) = 3(.4)(.6) = .72$$

4.92 We use the special formulas for the binomial mean and variance .

(a)

$$\mu = np = 555(.5) = 277.5$$

$$\sigma^2 = np(1-p) = 555(.5)(.5) = 138.75$$

(b)

$$\mu = np = 300(\frac{1}{6}) = 50$$

$$\sigma^2 = np(1-p) = 300(\frac{1}{6})(\frac{5}{6}) = 41.667$$

(c)

$$\mu = np = 700(.03) = 21$$

$$\sigma^2 = np(1-p) = 700(.03)(.97) = 20.37$$

4.93 We use the approximation $b(x; n, p) \approx f(x; np)$. Thus

$$f(2; 100(.02)) = f(2; 2) = 2^2 e^{-2}/2! = .2707.$$

4.94 (a) The mean is

$$\mu = 0(.5238) + 1(.4190) + 2(.0571) = .5332$$

and

$$\mu_2' = 0^2(.5238) + 1^2(.4190) + 2^2(.0571) = .6474$$

Thus

$$\sigma^2 = .6474 - (.5332)^2 = .3631.$$

(b) Using the special formulas for the hypergeometric distribution, we have:

$$\mu = n\frac{a}{N} = \frac{2 \cdot 4}{15} = .5333$$

$$\sigma^2 = \frac{na(N-a)(N-n)}{N^2(N-1)} = \frac{(2)(4)(11)(13)}{15^2(14)} = .3632$$

4.95 Since $(202 - 142)/12 = (142 - 82)/12 = 5$, we can apply Chebyshev's theorem with $k = 5$. Let X be the number of orders filled. Then,

$$P(X \leq 82 \text{ or } X \geq 202) = P(|X - 142| \geq 5 \cdot 12) \leq \frac{1}{25}$$

Thus,

$$P(82 < X < 202) > \frac{24}{25} = .96$$

4.96 $n = 10{,}000$, $p = .00004$, $np = .4$.

$$1 - F(1; .4) = 1 - (.938) = .062.$$

4.97 $\lambda = .6$ for two weeks. We need

$$f(0;6) = (.6)^0 e^{-.6}/0! = .5488.$$

4.98 For this problem, we need to use the multinomial distribution.

(a) The probability is given by:

$$f(2,1,0) = \frac{3!}{2!\ 1!\ 0!}\ (.8)^2\ (.15)^1\ (.05)^0 = .288$$

(b) To obtain the marginal distribution of X_2 we first calculate the probability that $X_2 = 0$. This is the sum of the probabilities

$$\begin{aligned} f_2(0) &= f(3,0,0) + f(2,0,1) + f(1,0,2) + f(0,0,3) \\ &= .512 + .096 + .006 + .0001 = .6141 \end{aligned}$$

Similarly,

$$\begin{aligned} f_2(1) &= f(2,1,0) + f(1,1,1) + f(0,1,2) = .288 + .036 + .0011 = .3251 \\ f_2(2) &= f(1,2,0) + f(0,2,1) = .0540 + .0034 = .0574 \\ f_2(3) &= f(0,3,0) = .0034 \end{aligned}$$

(c)

$$b(0;3,.15) = \binom{3}{0}\ (.15)^0\ (.85)^3 = .6141$$

$$b(1;3,.15) = \binom{3}{1}\ (.15)^1\ (.85)^2 = .3251$$

$$b(2;3,.15) = \binom{3}{2}\ (.15)^2\ (.85)^1 = .0574$$

$$b(3; 3, .15) = \binom{3}{3} (.15)^3 (.85)^0 = .0034$$

These agree exactly with the distribution in Part (b). They should because X_2 pertains to an event which occurs with probability .15 and does not occur with probability .8 + .05 = .85.

4.99 (a) The random numbers are distributed in the following table:

Number of spills	Probability	Cumulative probability	Random numbers
0	.2466	.2466	0000–2465
1	.3452	.5918	2466–5917
2	.2417	.8335	5918–8334
3	.1128	.9463	8335–9462
4	.0395	.9858	9463–9857
5	.0111	.9969	9858–9968
6	.0026	.9995	9969–9994
6	.0005	1.0000	9995–9999

(b) Using columns 9-12 of the third page of Table 7 and starting from row 101, gives:

Day	1	2	3	4	5
Random no.	8353	6862	0717	2171	3763
No. of spills	3	2	0	0	1

Day	6	7	8	9	10
Random no.	1230	6120	3443	9014	4124
No. of spills	0	2	1	3	1

Day	11	12	13	14	15
Random no.	7299	0127	5056	0314	9869
No. of spills	2	0	1	0	5

Day	16	17	18	19	20
Random no.	6251	4972	1354	3695	8898
No. of spills	2	1	0	1	3
Day	21	22	23	24	25
Random no.	1516	8319	3687	6065	3123
No. of spills	0	2	1	2	1
Day	26	27	28	29	30
Random no.	4802	8030	6960	1127	7749
No. of spills	1	2	2	0	2